

What are significant figures, what do they indicate, and how are they used in addition, subtraction, multiplication, and division?

The Model

There are two kinds of numbers in the world with respect to chemistry:

Exact Numbers	Inexact Numbers
• There are exactly 12 eggs in a dozen • Most people have exactly 10 fingers and 10 toes • 1 meter is exactly 100 centimeters • 1 yard is exactly 36 inches • 1 dollar is exactly 100 cents and exactly 4 quarters • 1 kilometer is exactly 1000 meters	• Any measured value. ○ Use of a 10 mL graduated cylinder to measure the volume of a solution might give a volume of 8.81 mL (3 significant figures) or a less precise volume of 8.7 mL with a 100 mL graduated cylinder. • An analytical balance might find the mass of a pencil to be 12.1403 g (6 SF), while a centigram balance might find it to weigh 12.13 g (4 SF).

The number of digits (significant figures) reported for a **measured value** conveys the quality of the measurement, and therefore, the quality of the measuring device. It is important to use significant figures correctly when reporting a measurement so that it does not appear to be more or less precise than the equipment used to make the measurement allows. We accomplish this by controlling the number of digits, or **significant figures**, used to report the measurement.

In this and other science courses, **you must use the correct number of significant figures when reporting your results!** Laboratory measuring instruments have their limits, just as your senses have their limits. One of your tasks, in addition to learning how to use various measuring instruments properly, will be to correctly determine the precision of the measuring devices that you use and to **report all measured and calculated values to the correct number of significant figures.**

Significant Figure Rules

There are three rules used to determine how many significant figures are in a number.

1. **Non-zero digits are always significant.** (See page 2 for details)
2. **Zeroes are significant when:** (See page 2 for details)
 - a. **They are in between two non-zero digits**
 - b. **They are at the end of a number that contains a decimal point**
3. **Zeroes are not significant when:** (See pages 2-3 for details)
 - a. **They are at the end of a number without a decimal point**
 - b. **They are at the beginning of a number**

Focus on these rules and learn them well! They will be used extensively in class. Please practice and practice as you need to be solid on the concept of significant figures.

Please remember that, in science, with the exception of a few well defined, exact numbers, all numbers are based on measurements. Since all measurements are uncertain, you must only use those numbers

that are meaningful. A common ruler can't measure something to be 22.0014224 cm long. Not all digits have meaning (significance) and therefore are not written down. Only numbers that meet the significant figure rules are reported.

Rule 1: Non-zero digits are always significant

Hopefully, this rule seems rather obvious! If you measure something and the device you use (ruler, thermometer, balance, graduated cylinder) returns a number to you, then you have made a measurement decision and that ACT of measuring gives significance to that particular number in the overall value you obtain.

Thus, if you measure a volume of 24.22 mL on a buret, you would have 4 significant figures. If your balance gave you a mass of 1.44 g, you would have 3 significant figures.

Rule 2: Zeroes are significant when:

They are in between a non-zero digit. (think 'sandwiched' zeroes)

Suppose you had measured a value like 406. According to the first rule, the 4 and the 6 are significant. However, to make a decision on the 4 (hundreds place) and the 6 (ones place), you **HAD** to have made a decision on the tens place. The measurement scale for this number would have calibration marks for the hundreds and tens places with an estimation made in the 'ones' place. **Thus, significant figures indicate the number of digits known with certainty (the first two digits in 406) and one digit that is an estimate (the 6 in 406).** Such a measurement scale would look like this:

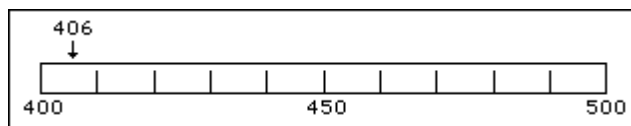


Figure 1. A measuring scale that allows for 3 significant figures

They are at the END of a number with a decimal point. (Certain trailing zeroes)

These zeroes are in the 'estimated' position for a number. For example, examples where a trailing zero is significant are: (the significant zeroes are in **BOLD**)

$$\begin{array}{c} 12.7\mathbf{0} \\ 0.0035\mathbf{0} \\ 1.25\mathbf{0} \times 10^6 \end{array}$$

In each case, all of the non zeroes count (rule number 1) – they are numbers that are known with certainty. In each case, the zeroes at the end of the numbers are the estimated number – the number that is at the limit of reliability on the instrument you are using.

Rule 3: Zeroes are NOT significant when:

They are at the end of a number without a decimal point. These are simply placeholders.

200 has only **one** significant figure

25,000 has two significant figures

In both cases only the **bolded** numbers are significant. In 200, the 2 is both the known and estimated digit, in 25,000 the 2 is the known digit and the 5 is the estimated digit. Both numbers can also be written in scientific notation to get rid of the zeroes.

200 can be 2.0×10^2

25,000 can be 2.5×10^4

In both cases, the zeroes are holding the place

How do you know how many significant figures are in a number like 200? In a problem without a measurement context, you will be told. If you are performing an experiment, the context of the experiment and the measuring device will tell you how many digits to report.

They are at the beginning of a number (Leading zeroes). They are simply placeholders.

The two numbers below are examples of this rule.

0.00**546** this number has three significant figures

0.00**3054** this number has four significant figures

The leading zeroes in each case have only 1 function – to locate the decimal point. They are not involved in a measurement decision. The leading zeroes disappear when these are written in scientific notation.

5.46×10^{-3}

3.054×10^{-3}

Exact Numbers

Exact numbers, such as the number of people in this room, have an infinite number of significant figures. Exact numbers are count up how many of something are present; they are not measurements made with instruments. Another example of this are defined numbers, such as 1 foot=12 inches. There are exactly 12 inches in one foot. If a number is exact, it DOES NOT IMPACT THE PRECISION OF THE CALCULATION.

Other examples:

100 years = 1 century

1 g = 1000 mg

H₂O is made of 2 atoms of hydrogen and one atom of oxygen

3.785 L = 1 gallon

A note about texts – There might be times when your text will not be clear on the number of significant figures. Suppose you have a problem that says 100 mL. There is only one significant figure in 100 and this seems odd for a text. By using a graduated cylinder, you can measure to 3 places easily, usually 4 places. Why then, does the book only use 1 and not write 100., or 100.0, or 1.00×10^2 ? The writers probably assume that you understand this and will use 3 or 4 significant figures. So... if you come across a problem like this, use 3 or 4 significant figures. Change it to 100., or 100.0. These are reasonable for chemistry.

Key Questions

1. What kind of numbers are exact numbers? Give at least one **original** example.
2. What kind of numbers are inexact numbers? Why? Give at least one **original** example.

3. What is your understanding of significant figures: what are they, when should they be used, and what function do they serve?

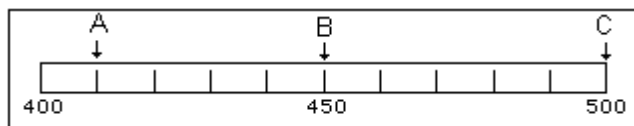


Figure 2. A hypothetical measuring scale

4. What values would you record for measurements A, B, and C if each measurement fell on the line each arrow points to in figure 2? How many sig figs should each measurement have?
- A = _____ B = _____ C = _____
5. Later in the class you will be asked to use an analytical balance that measures to the 0.0001 place. If you need 3 grams of a solid, what mass would you measure using the balance and how would you report your number?
6. Suppose you are asked to measure out about 25 mL of water as accurately as you can.
- What measuring device would you use?
 - How much water should you measure out?
 - How many significant figures would you report?
7. How many significant figures are there in each of the following numbers? Record your responses in the spaces provided and circle the digits in the original number that are significant.

- | | |
|---------------------------------|-------------------------------|
| a.) 3.0800 _____ | f.) 3.200×10^9 _____ |
| b.) 0.004218 _____ | g.) 340 _____ |
| c.) 7.09×10^{-5} _____ | h.) 780,000,000 _____ |
| d.) 91,900 _____ | i.) 0.001001 _____ |
| e.) 0.003005 _____ | j.) 500.0700 _____ |

Rounding numbers – follow the simple, “if it’s 5 or higher, round up, less than 5 leave alone” rule.

8. Round the following numbers to four significant figures.
- | | |
|-----------------------------------|---------------------|
| a.) 2.16347×10^5 = _____ | d.) 7.1214 = _____ |
| b.) 4.000574 = _____ | e.) 375.689 = _____ |

The Model – Using Significant Figures in Addition and Subtraction

Did you know that $20,000 + 1$ does not always equal 20,001? In fact, $20,000 + 1$ is sometimes equal to 20,000. Here's why:

Remember that zeroes in a number are NOT always *significant*. Knowing this makes a big difference in how we add and subtract. For example, consider a swimming pool that can hold 20,000 gallons of water. If I fill the pool to the maximum fill line and then go and fill an empty one gallon milk jug with water and added it to the pool, do I have exactly 20,001 gallons of water in the pool? Of course not. I had approximately 20,000 gallons before and after I added the additional gallon because "20,000 gallons" is not a very precise measurement.

In mathematical operations involving significant figures, the answer is reported in a way that reflects the reliability of the **least precise (reliable) number**. **An answer is no more precise than the least precise number used to get the answer.**

Use the "Decimal rule" when adding and subtracting numbers.

For addition and subtraction, the answer must be rounded off to contain only as many decimal places as are in the value with the least number of decimal places.

As you'll see next, this can be viewed as lining up the decimals and rounding your answer to the least precise position. PLEASE NOTE: the rules for addition and subtraction are different from those for multiplication/division.

Example 1. $350.04 + 720 = 1070.04 = 1070$

The answer can only be as precise as the LEAST precise number in the operation. Therefore the answer must be rounded off to the tens place since the tens place is less precise than the hundredths place.

$$\begin{array}{r} 350.04 \\ + 720 \\ \hline 1070.04 \\ 1070 \end{array}$$

← This number is precise to the hundredths place

← This number is precise to the tens place

Rounded, 0 is less than 5

Example 2. $7000 - 1770 = 5230 = 5000$

The answer can only be as precise as the LEAST precise number in the operation. Therefore the answer must be rounded off to the thousands place since the thousands place is less precise than the tens place.

$$\begin{array}{r} 7000 \\ - 1770 \\ \hline 5230 \\ \hline 5000 \end{array}$$

This number is precise to the thousands place

This number is precise to the tens place

Rounded, 2 is less than 5

Key Questions

9. Consider example #1 above. In the spaces below, indicate the number of significant figures (sf) for each number in the problem.

350.04
_____sf

720
_____sf

1070.04
_____sf

1070
_____sf

Should the number of significant figures be considered when added or subtracting measured numbers? Explain your reasoning.

10. When you add and subtract numbers, how do you identify the first uncertain number in the result?

Exercises

Record the answer before and after rounding for each problem below.

14. $3.461725 + 14.91 + 0.980001 + 5.2631 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

15. $23.1 + 4.77 + 125.39 + 3.851 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

16. $22.101 - 0.9037 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

17. $0.014046 - 0.0004450 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

18. $564,321 - 264,321 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

The Model – Using Significant Figures in Multiplication and Division

A chain is not stronger than its weakest link – that is, an answer is no more precise than the least precise number used to get the answer. Use this concept when multiplying and dividing numbers.

*When measurements are multiplied or divided, the answer can contain no more significant figures than the number with the fewest number of significant figures. This means you **MUST** know how to recognize significant figures in order to use this rule.*

To round correctly, follow these simple steps

1. Count the number of significant figures in each number
2. Round your answer to the least number of significant figures.

Example 1:

$$\begin{array}{c} \nearrow \quad \quad \quad \nearrow \quad \quad \quad \nwarrow \\ 3 \text{ sig figs} \quad 13.1 * 1.2039 = 15.77109 = 15.8 \quad 5 \text{ sig figs} \end{array}$$

The rounded answer has 3 significant figures since 3 is the least number of significant figures in this problem

Example 2:

$$\begin{array}{c} \nwarrow \quad \quad \quad \nwarrow \quad \quad \quad \nwarrow \\ 3 \text{ sig figs} \quad 4560 / 14 = 325.714285714 = 330 \quad 2 \text{ sig figs} \end{array}$$

The rounded answer has only 2 significant figures since 2 is the least number of significant figures in this problem.

Multi-Step Calculations: Keep at Least One Extra Significant Figure in Intermediate Answers

When doing multi-step calculations, *keep at least one more significant figure in intermediate results* than needed in your final answer. For example, if a final answer requires two significant figures, then carry *at least* three significant figures in all calculations. If you round-off all your intermediate answers to only two digits, you are discarding the information contained in the third digit, and as a result the *second* digit in your final answer might be incorrect. This phenomenon is known as "round-off error." *Avoid rounding errors by carrying at least one extra sig fig throughout a multi-step calculation and then round off to the correct number of sig figs at the very end.*

Key Questions

19. When you multiply and divide numbers, what is the relationship between the number of significant figures in the result and the number of significant figures in the numbers you are multiplying or dividing?

Exercises

Record the answer before and after rounding off for each problem below.

20.) $(3.4617 \times 10^7) \div (5.61 \times 10^{-4}) = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

21.) $[(9.714 \times 10^5) (2.1482 \times 10^{-9})] \div [(4.1212) (3.7792 \times 10^{-5})] = \underline{\hspace{2cm}}$

(Watch your order of operations on this problem!) $= \underline{\hspace{2cm}}$

22.) $(4.7620 \times 10^{-15}) \div [(3.8529 \times 10^{12}) (2.813 \times 10^{-7}) (9.50)] = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

23.) $[(561.0) (34,908) (23.0)] \div [(21.888) (75.2) (120.00)] = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

24. Carry out each of the following calculations. Check that each answer has the correct number of significant figures and the correct units of measure.

a.) $\frac{2.420 \text{ g} + 15.6 \text{ g}}{4.8 \text{ g}} =$

b.) $\frac{7.87 \text{ g}}{16.1 \text{ mL} - 8.44 \text{ mL}} =$

c.) $V = \pi r^2 h$ where $r = 6.23 \text{ cm}$ and $h = 4.630 \text{ cm}$

$V =$

$$\text{d.) } \frac{8.32 \times 10^7 g}{\frac{4}{3}(3.1416)(1.95 \times 10^2 \text{ cm})^3} =$$

Note: 4/3 is an exact number!

$$\text{e.) } E_k = \frac{1}{2}mv^2 = \frac{(1.84 \times 10^2 g)(44.7 \text{ m/s})^2}{2} =$$

$$\text{f.) } \frac{(1.07 \times 10^{-4} \frac{\text{mol}}{\text{L}})^2 (2.6 \times 10^{-3} \frac{\text{mol}}{\text{L}})}{(8.35 \times 10^{-5} \frac{\text{mol}}{\text{L}})(1.48 \times 10^{-2} \frac{\text{mol}}{\text{L}})^3} =$$