

Calculating Acceleration

Ex- z - displacement in the " z " direction. v = velocity, etc...

In uniformly accelerated motion, velocity changes uniformly over time. Reworking $\mathbf{a} = \Delta \mathbf{v} / \Delta t$ gives $\Delta \mathbf{v} = \mathbf{a} \Delta t$. For a freely falling body, velocity will change by 9.8 m/s each second. For a ramp with constant acceleration of 0.5 m/s², a ball initially rolling at 2 m/s will roll at 2.5 m/s after 1 s on the ramp, 3 m/s after 2 s, and so on.

Non-accelerating motion	Accelerating motion
$x_f = x_i + v_{i,x}t$ $y_f = y_i + v_{i,y}t$	$x_f = x_i + v_{i,x}t + (1/2) a_x t^2$ $y_f = y_i + v_{i,y}t + (1/2) a_y t^2$

Because projectile motion can be broken into horizontal and vertical components, the equations can be used to analyze each component separately. A ball that rolls off a table with an initial horizontal velocity will have a horizontal displacement and velocity described by the formulas for non-accelerating motion. The vertical aspect of the projectile's motion is described with the formulas for accelerating motion.

Horizontal motion of projectiles	Vertical motion of projectiles
$x_f = x_i + v_{i,x}t$ $v_{f,x} = v_{i,x}$	$y_f = y_i + v_{i,y}t + (1/2) g t^2$ $v_{f,y} = v_{i,y} + g t$

For projectiles moving under the influence of gravity, the horizontal velocity will be constant. The acceleration due to gravity is at a right angle to the direction of motion and so will not change the horizontal speed. The vertical component of velocity will change at a constant rate, $g = -9.8 \text{ m/s}^2$. The vertical displacement at time t is a result of three factors: the starting point y_i , the displacement due to initial velocity $v_{i,y}t$, and the displacement due to acceleration $(1/2)gt^2$.

Compare the formulas in the two tables. They apply to the general cases of constant velocity or constant acceleration in any direction. A runner who accelerates for the last stretch of the race at 1.5 m/s² could go from a horizontal speed of 12 m/s to 15 m/s in 2 seconds. A weight sinking through water at a constant downward speed of 0.3 m/s will descend 1.5 meters in 5 seconds.

EXPLAIN How can scientists model the motion of a projectile to analyze motion as the sum of more easily understood parts and to make predictions about future motion given initial conditions?



Evidence Notebook Refer back to Figure 12. Analyze the motion shown in the photograph by considering the horizontal and vertical components separately. How does this motion compare to the skier's motion? Sketch an analysis of the skier's motion similar to that in Figure 12b. How are one-dimensional and two-dimensional motion related?



Hands-On Lab

Measuring Acceleration

Suppose you are studying the motion of a ball across a table, but you don't have sophisticated equipment to measure motion. How can you use projectile motion to measure the object's acceleration down a ramp?

MAKE A CLAIM

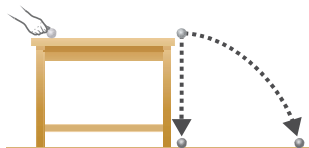
In Figure 20a, one ball falls straight down, and one flies out as a projectile. How do you expect their landing times to compare?



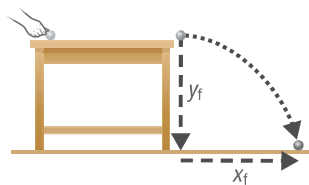
safety goggles



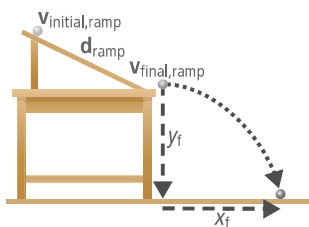
FIGURE 20: A ball rolls off a level table. The path followed by the projectile is defined by y_f and x_f . In figure c, the ball is launched using a ramp, making it easy to reproduce the same horizontal velocity in each trial.



a Trial 1



b Trial 2



c Trial 3

MATERIALS

- safety goggles
- desk or table
- flat surface that can be used to make a ramp, such as a binder
- meterstick
- two identical balls that can hit the floor without damage

SAFETY INFORMATION

- Wear safety goggles during the setup, hands-on, and takedown segments of the activity.
- Immediately pick up any items dropped on the floor so they do not become a slip/fall hazard.

CARRY OUT THE INVESTIGATION

Calculate Horizontal Velocity

Trial 1: Roll a ball off the edge of a table. At the instant the rolling ball leaves the table, drop a second ball from the same height. Do the two balls land at the same time? Is the result what you expect after studying parabolic motion?

Trial 2: Vary the speed at which you roll the first ball off the table. Observe what happens. For one trial, measure the horizontal distance from the table edge to where the first ball lands, x_f . Measure the vertical distance both balls fall, y_f . You will use these values to calculate the time the ball takes to fall and the initial horizontal velocity when the first ball is launched.

The initial vertical velocity for both balls is zero, since the first ball is moving horizontally and the second ball is at rest. Use the following method to calculate the initial horizontal velocity for the first ball.

1. Use your value for y_f and the equation $y_f = (1/2)(9.8 \text{ m/s}^2)t^2$ to find t , the time the ball took to fall.
2. Substitute this value for t and the value you measured for x_f to solve $x_f = 0 + v_x t$ for v_x .

You can use this technique to calculate the horizontal velocity of a ball that is launched across the table by hand, with a spring, or from a ramp.

Calculate the Acceleration of a Ball on a Ramp

Trial 3: Roll the first ball down a ramp that is set back from the edge of the table by a short distance. The ball should be moving horizontally when it gets to the edge of the table and falls off. Use the following method to calculate the acceleration of the ball:

1. Measure the length of the ramp, x , and substitute your value into the equation for accelerated motion, $x_f = x_i + v_i t + (1/2)at^2$. Both x_i and v_i are 0, so this will reduce to $x = (1/2)at^2$. The values of a and t are still unknown.
2. Substitute $(\Delta v/t)$ for a :
$$x = (1/2)at^2$$
$$x = (1/2)(\Delta v/t)t^2$$
$$x = (1/2)\Delta v t$$
3. The horizontal velocity for the launched ball is also the ball's final velocity when it reaches the bottom of the ramp. Use the method from Trial 2 to calculate Δv for the ramp.
4. Substitute your values for x and Δv to solve for t .
5. Then solve $a = (\Delta v/t)$ for a .

ANALYZE

1. What evidence do you have that motion can be separated into horizontal and vertical motion?

2. How does the horizontal speed of the ball rolling across the table correspond to x_f ?

3. How does the change in velocity imparted by the ramp correspond to its length?

EXTEND

1. Suppose the ball is launched by a hand giving it a steady push over a certain distance. How would you calculate the average acceleration of that push?

2. What are some other methods and equipment that could be used to measure the acceleration of an object? What are the possible advantages of the other methods?
