

$$1. a) \cot^2(\theta) - \cos^2(\theta) = \cot^2\theta \cos^2\theta$$

Let consider LHS.

$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} \text{ (substitute into RHS)}$$

$$\Rightarrow \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$$

$$= \cos^2\theta \left(\frac{1}{\sin^2\theta} - 1 \right)$$

$$= \cos^2\theta \left(\frac{1 - \sin^2\theta}{\sin^2\theta} \right)$$

$$\therefore 1 - \sin^2\theta = \cos^2\theta$$

$$= \cos^2\theta \times \left(\frac{\cos^2\theta}{\sin^2\theta} \right)$$

$$= \cos^2\theta \cot^2\theta \text{ (RHS)}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$6). \frac{1 + \cos t}{\sin t} = \frac{\sin t}{1 - \cos t}$$

Let's take RHS.

$$= \frac{\sin t}{1 - \cos t} \times \frac{1 + \cos t}{1 + \cos t}$$

$$= \frac{\sin t (1 + \cos t)}{(1 - \cos t)(1 + \cos t)}$$

$$= \frac{\sin t + \sin t \cos t}{1 - \cos^2 t} \Rightarrow \frac{\sin t + \sin t \cos t}{\sin^2 t}$$

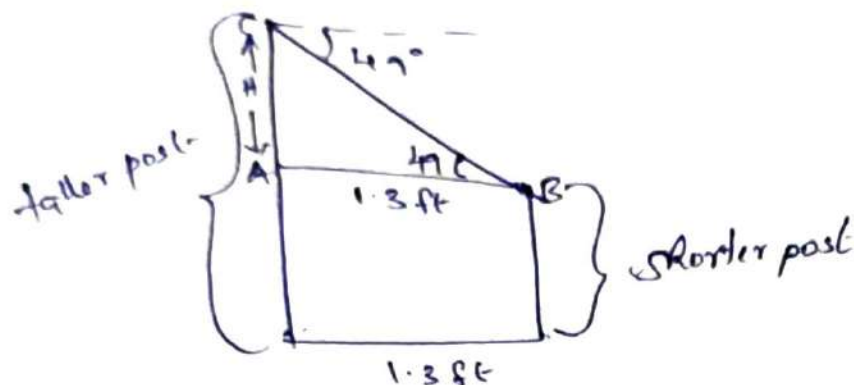
$$= \frac{\sin t (1 + \cos t)}{\sin^2 t} \Rightarrow \frac{1 + \cos t}{\sin t} \text{ (LHS)}$$

$$\therefore \text{RHS} = \text{LHS}$$

2. The focus are separated by a distance

of 1.3 ft

The angle of depression from the top of taller to shorter post is 49° .



In $\triangle CAB$

$$\tan 49^\circ = \frac{H}{1.3}$$

$$H = 1.3 \tan 49^\circ$$

$$H = 1.495 \text{ ft}$$

$$H \approx 1.5 \text{ ft}$$

The shorter post should be raised to 1.5 ft to match the height of the taller post.

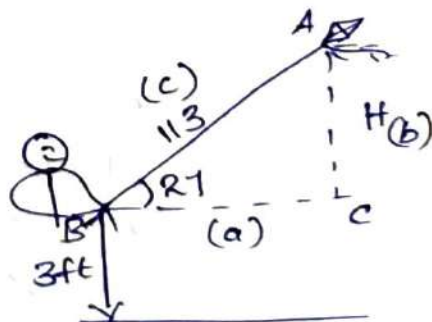
3. We have the total height $y = b + 3 \text{ ft}$

$$\sin \theta = b/c, \quad H = b$$

$$b = c \sin \theta$$

$$y = 113 \sin 27^\circ + 3 \text{ ft}$$

$$y = 54.30 \text{ ft}$$



Extra credit

1. $(\sin \beta + \cos \beta)^2 - 2 \sin(\beta) \cos(\beta) = 1$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

Take LHS

$$= \sin^2 \beta + \cos^2 \beta + 2 \sin \beta \cos \beta - 2 \sin \beta \cos \beta$$

$$= \sin^2 \beta + \cos^2 \beta$$

$$= 1$$

$$\therefore \text{LHS} = \text{RHS}.$$

2. $\ln(\sec y) = -\ln(\cos y)$

$$\sec y = \frac{1}{\cos y}$$

LHS

$$= \ln\left(\frac{1}{\cos y}\right)$$

$$= \ln(1) - \ln(\cos y)$$

$$= 0 - \ln(\cos y)$$

$$= -\ln(\cos y)$$

$$\therefore \text{LHS} = \text{RHS}.$$

$$\therefore \ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

3. Despite the fact that blazing stars and galaxies shine throughout the universe, space is pitch black rather than being brightly lit.