

1. (a) A company uses resources A and B to manufacture three components, amplifiers, modulators and oscillators. These components are then combined into products α and β .

There are 600 units of resource A and 500 units of resource B available each day. One amplifier requires 3 units of resource A and 3 units of resource B. One modulator requires 4 units of resource A and 5 units of resource B. One oscillator requires 4 units of resource A and 2 units of resource B.

One unit of product α requires one amplifier, two modulators and three oscillators. One unit of product β requires three amplifiers, two modulators and five oscillators.

One unit of product α sells for a profit of £10 and one unit of product β sells for a profit of £13.

Write a linear programming problem to decide how many of each component and each product to make each day. You must explain any notation you introduce, and explain your constraints and objective function.

(Do not attempt to solve the linear programming problem. You may ignore any need to produce whole numbers of items.) [10 marks]

- (b) The situation is actually a bit more complicated. The components are created at a component manufacture site, and the products are created at separate product manufacture (PM) sites. Product α is made at PM site α and product β is made at PM site β .

The components must be sent to the product manufacture sites, and this incurs a transportation cost. Each amplifier costs £1 to send to PM site α , and £1 to send to PM site β . Each modulator costs £4 to send to PM site α and £2 to send to PM site β . Each oscillator costs £2 to send to PM site α and £1 to send to PM site β .

Write a linear programming problem to address this new problem. You may assume that sending components to sites is instantaneous. You must explain any notation you introduce, and explain your constraints and objective function.

(Do not attempt to solve the linear programming problem. You may ignore any need to produce whole numbers of items.) [5 marks]

- (c) Suppose for this part that all transportation costs are zero.

- (i) Without solving the LPPs, answer the following question. Is the optimal value of your LPP in (a) less than, greater than, or equal to the optimal value of your LPP in (b)? Justify your answer mathematically. [2 marks]
- (ii) How does your answer to (i) change if we require that whole numbers of components and products are produced? Justify your answer mathematically. [3 marks]