1. Yes, when the null hypothesis tests false that hints that the alternative hypothesis has a chance of being true.

2. (i) State the null hypothesis and alternative hypothesis
(ii) Identify type of test to be performed ~~Calculate test statistics~~
(iii) Specify alpha level
(iv) Obtain critical values and make a decision to reject or accept the null hypothesis

3. $\mu = 20$

a) Null hypothesis : The course did not affect the scores
Alternative hypothesis : The course affected the scores

b) $H_0 : \mu = 20$
$H_1 : \mu \neq 20$

4. Alpha level is the level of committing a mistake of rejecting the null hypothesis when it is true.

Critical region is a set of all values that will cause us to reject the null hypothesis.

5. Type I error ⇒ mistake of rejecting H_0 when it is true

Type II error ⇒ Mistake of accepting H_0 when it is false
Type I error is worse than Type 2 error because type I error is false positive.

6. $\alpha = 0.05$ to 0.01
a) The boundaries are pushed outwardly
b) Probability decreases.

7 (a) It increases z-score.
(b) This reduces the z-score
(c) This reduces the z-score

8. $\mu = 72$
$\sigma = 12$, $n = 64$
$M = 76$

Null hypothesis: The implant does not change life expectancy
Alternative hypothesis: The implant changes life expectancy

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{76-72}{12/\sqrt{64}} = 2.667$$

$$\alpha = 0.05/2 = 0.025$$

Z from tables $= \pm 1.96$

Z values are in the rejection region therefore we lack enough evidence to accept the null hypothesis. Therefore the implant changes life expectancy.

9. $\mu = 15$, $\sigma = 9$, $n = 36$
$M = 12$

a) $H_0: \mu = 15$ The program did not increase the hours spend in preparing for class
$H_1: \mu \neq 15$ The programme increased the hours spend in preparing for class.

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{12-15}{9/\sqrt{36}} = -2$$

$\alpha = 0.05/2 = 0.025$
confidence level $= 1 - 0.025 = 0.975$
Z table value $= \pm 1.96$
-2 is in the rejection region, therefore we reject H_0.

10. $n = 16$ $\sigma = 1.05$

$M = 4.50$ $\alpha = 0.05$

$\mu = 4.24$

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{4.50 - 4.24}{1.05/\sqrt{16}} = 0.9904$$

$\alpha = 0.05/2 = 0.025$

Confidence level $= 0.975$

$Z_{table} = \pm 1.96$

Therefore we accept Ho since Z value is in the acceptance region

11. $n = 16$

$M = 72.5$

$\mu = 77$, $\sigma = 8$

a) Null: Students studying from an electronic screen had exam scores that are not significantly different from the general population $H_0: \mu = 77$

Alternative: Students studying from a screen had significantly different scores from the general population, $H_1: \mu \neq 77$

b) $H_0: \mu = 77$

$H_1: \mu \neq 77.$

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{72.5 - 77}{8/\sqrt{16}} = -2.25$$

$\alpha = 0.05/2 = 0.025$

Confidence level $= 1 - 0.025 = 0.975 \Rightarrow Z = \pm 1.96$

Z value is beyond the acceptance region ie in the rejection region ∴ We reject the Ho

12. $n = 100$

$\mu = 50$, $\sigma = 15$

$M = 53.8$, $\alpha = 0.05$

$H_0 : \mu = 50$

$H_1 : \mu \neq 50$

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{53.8 - 50}{15/\sqrt{100}} = 2.5333$$

$$\alpha = 0.05/2 = 0.025$$

confidence level $= 0.975$

$Z_{table} = \pm 1.96$

The z value falls ~~within~~ outside the acceptance region and therefore we ~~accept~~ reject Ho.

b) $\sigma_m = \sigma/\sqrt{n} = 15/\sqrt{100} = 1.05$

c) Outcome: There is no significant difference between the scores

13. $\sigma = 10$

$\mu = 20$

$M = 25$

a) $n = 25$

$H_0 : \mu = 20$

$H_1 : \mu \neq 20$

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{25 - 20}{10/\sqrt{25}} = +2.5$$

$\alpha = 0.05 \Rightarrow Z = \pm 1.96$

Since Z is in the rejection region, we reject Ho and conclude that there is significant difference.

(b) $n = 4$

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{25 - 20}{10/\sqrt{4}} = +1$$

$$\alpha = 0.05 \Rightarrow Z = \pm 1.96$$

We accept Ho and conclude that there is no significant difference

c) Size of the sample could lead us to accept or reject a null hypothesis. When the size decreases we end up accepting the hypothesis. When it increases we reject Ho.

14. $n = 9$

$\mu = 100$

$M = 106$, $\sigma = 10$

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{106 - 100}{10/\sqrt{9}} = \frac{6}{10/3} = 6 \times 3/10 = 1.8$$

$$Z_{0.05/2} = Z_{0.025} = \pm 1.96.$$

a) Z stat falls in the acceptance region and therefore we lack enough evidence to reject Ho

b) Z score $= 1.8$

$Z_{0.05} = 1.645$

Our z score falls in the rejection rejection and we therefore lack enough evidence to accept Ho.

c) $\sigma = 12$

$\alpha = 0.05$

$$Z_{score} = \frac{106 - 100}{12/\sqrt{9}} = 1.5$$

$Z_{crit} = \pm 1.96$

We therefore lack enough evidence to reject Ho and thus there is no significant effect.

Z score = 1.5

Z critical = 1.645

we still lack enough evidence to reject Ho and conclude that there is no significant effect.

for a and b, when number of tails decrease, we end up getting a different conclusion..

For c and d, when the standard deviation increases, the Z score falls in the acceptance region. When the tails are reduced when the score is in a safe location in the acceptance region, no change on our decision as in part d.

$\mu = 40$

$\sigma = 10$

$M = 46$

a) $Z_{0.025} = \pm 1.96$

Z score should be at least ~~as~~ equal to 1.96

$$Z = \frac{M-\mu}{\sigma/\sqrt{n}}$$

$$\frac{Z\sigma}{\sqrt{n}} = M-\mu \Rightarrow \sqrt{n} = \frac{Z\sigma}{M-\mu}$$

$$n = \left(\frac{Z\sigma}{M-\mu}\right)^2$$

$$= \left(\frac{1.96 \times 10}{46-40}\right)^2 = \left(\frac{19.6}{6}\right)^2 = 10.67 = 11$$

(b) $M = 43$

$$n = \left(\frac{Z\sigma}{M-\mu}\right)^2 = \left[\frac{1.96 \times 10}{43-40}\right]^2 = \left[\frac{19.6}{3}\right]^2 = 42.68 = 43$$

16. $\mu = 9.6$

$\sigma = 1.9$

$n = 4$ $\quad H_0 : \mu = 9.6$

$M = 12.25$ $\quad H_1 : \mu \neq 9.6$

$$Z \text{ score} = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{12.25 - 9.6}{1.9/\sqrt{4}} = 3.3125$$

Z score $= 1.645$

We reject the null hypothesis that the past 4 years had no more 90° fahrenhait days and conclude that the past 4 years had more 90° fahreheit days than would be expected.

17. $n = 20$

$M = 562$

$\mu = 500$

$\sigma = 100$

a) $H_0 : \mu = 500$

$H_1 : \mu > 500$

$$Z \text{ score} = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{562 - 500}{100/\sqrt{20}} = 2.772$$

$Z_{0.01} = 2.326$

Z score lies in the rejection region, we reject H_0 and therefore we conclude that the SAT scores of the students were higher than the general population

(b) $$\text{Cohen's } d = \frac{|M - \mu|}{\sigma} = \frac{|562 - 500|}{100} = 0.62$$

(c) We lack enough evidence to accept Ho and therefore we conclude that the SAT scores of the treated group were higher than the general population.

cohen's d depicts a medium effect since it is 0.62.

18. $\mu = 2.06$ $\sigma = 1.00$

$n = 25$

$M = 1.66$

$$Z = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{1.66 - 2.06}{1.00/\sqrt{25}} = -2$$

a)

$H_0 : \mu = 2.06$

$H_1 : \mu < 2.06$

$Z_{0.05} = -1.645$

-2 lies in the rejection region and therefore we reject Ho and conclude that the treatment reduced depression.

b) decrease the significance level

c) $\text{Cohen's } d = \frac{|M - \mu|}{\sigma} = \frac{|1.66 - 2.06|}{1.00} = 0.4$

(d) Z score lies in the rejection rejection and therefore we reject Ho and conclude that the treatment reduced depression symptoms. Our value of cohen's d depicts a medium effect.

19. Yes, as long as the conditions for a Z-test are met. Such a variation cancels the effect and keeps Z score in check.

20. $n = 25$

$M = 7.29$

$\mu = 7.52$ $\sigma = 0.60$

$H_0 : \mu = 7.52$

$H_1 : \mu < 7.52$

$$Z \text{ score} = \frac{M - \mu}{\sigma/\sqrt{n}} = \frac{7.29 - 7.52}{0.60/\sqrt{25}} = -1.91667$$

$Z_{crit} = -1.645$, our value of z lies in the rejection region therefore reject Ho and uphold H_1

(b) Cohen's d $= \frac{|M - \mu|}{\sigma} = \frac{|7.79 - 7.52|}{0.60} = 0.3833$

(c) Our z score lies in the rejection region and therefore we reject Ho and conclude that the review ratings are lower than that of general population on a hot weather.

Cohen's d tells us there is a small effect

21. Finding the upper tail $b = \mu + z \cdot \frac{\sigma}{\sqrt{n}} = 55$

~~$b = 50 + 1.96 \cdot \frac{10}{\sqrt{4}} = 59.8$.~~

$z \text{ score} = \frac{b - \mu}{\sigma/\sqrt{n}} = \frac{\overset{55}{\cancel{59.8}} - 50}{10/\sqrt{4}} = 1.0$

That corresponds to a P value of 0.15866 which gives us a power of $(1-P) = (1 - 0.1587) = 0.84134 = 0.84134$

(b) $n = 25$

$$b = \mu + z \cdot \frac{\sigma}{\sqrt{n}} = \frac{55}{\sqrt{25}}$$

$$Z = \frac{b-\mu}{\sigma/\sqrt{n}} = \frac{55 - 50}{10/\sqrt{25}} = 2.5$$

P value $= 0.0062$

Power $= 1 - P = 0.9938 = 0.9938$

22. $n = 4$

$\mu = 50 \quad \sigma = 8$

Upper limit $b = 50 + 3 = 53$.

a)

$$Z \text{ score} = \frac{b-\mu}{\sigma/\sqrt{n}} = \frac{53-50}{8/\sqrt{4}} = 0.75$$

P value $= 0.22663$.

power $= 1 - P$ value $= 1 - 0.22663 = 0.77337$.

b) $b = 53$

$$Z \text{ score} = \frac{b-\mu}{\sigma/\sqrt{n}} = \frac{53-50}{8/\sqrt{64}} = \frac{3}{1}$$

P value $= 0.00135$

statistical power $= 1 - 0.00135 = 0.99865$