

Problem 1.

- i. Expanding $Var(\hat{\beta}_1 - 3\hat{\beta}_2)$,

$$Var(\hat{\beta}_1 - 3\hat{\beta}_2) = Var(\hat{\beta}_1) - 6 Cov(\hat{\beta}_1, \hat{\beta}_2) + 9 Var(\hat{\beta}_2)$$

The standard error (se) is the square of root of the variance

$$se(\hat{\beta}_1 - 3\hat{\beta}_2) = \sqrt{Var(\hat{\beta}_1 - 3\hat{\beta}_2)}$$

$$se(\hat{\beta}_1 - 3\hat{\beta}_2) = \sqrt{Var(\hat{\beta}_1) - 6 Cov(\hat{\beta}_1, \hat{\beta}_2) + 9 Var(\hat{\beta}_2)}$$

- ii. t-statistics for testing the hypothesis $H_0 : \beta_1 - 3\beta_2 = 1$

$$H_0 : \beta_1 - 3\beta_2 - 1 = 0$$

$$t = \frac{\hat{\beta}_1 - 3\hat{\beta}_2 - 1}{se(\hat{\beta}_1 - 3\hat{\beta}_2)}$$

$$t = \frac{\hat{\beta}_1 - 3\hat{\beta}_2 - 1}{\sqrt{Var(\hat{\beta}_1) - 6 Cov(\hat{\beta}_1, \hat{\beta}_2) + 9 Var(\hat{\beta}_2)}}$$

- iii. Regression equation involving $\beta_0, \theta_0, \beta_2$ and β_3

$$\text{Since } \theta_1 = \beta_1 - 3\beta_2$$

$$\text{Then by rewriting } \beta_1 = \theta_1 + 3\beta_2$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \mu$$

Replacing

$$y = \beta_0 + (\theta_1 + 3\beta_2)x_1 + \beta_2x_2 + \beta_3x_3 + \mu$$

Expanding

$$y = \beta_0 + \theta_1 x_1 + 3\beta_2x_1 + \beta_2x_2 + \beta_3x_3 + \mu$$

$$y = \beta_0 + \theta_1 x_1 + \beta_2(3x_1 + x_2) + \beta_3x_3 + \mu$$

Thus the regression equation, regress y on x_1 , $(3x_1 + x_2)$ and x_3

Problem 2.

Using

$$plim(\hat{\beta}_1) = \beta_1 + \beta_2\delta_1$$

$$\text{where } \delta_1 = \frac{Cov(x_1, x_2)}{Var(x_1)}$$

A higher tolerance of risk means more willingness to invest in the stock market, so

$$\beta_2 > 0$$

By assumption, funds and risktol are positively correlated.

$$\delta_1 > 0 : plim(\hat{\beta}_1) = \beta_1 + \beta_2\delta_1 > \beta_1$$

Hence $\hat{\beta}_1$ has a positive inconsistency (asymptotic bias)

If we drop risktol from the regression equation which is positively correlated with funds, some of the estimated effect of funds is actually due to the effect of risktol.

The estimator will be greater than the true value of the population parameter.