1 $\frac{dy}{dx} = -7xy \quad , y(0) = 6$

$$\frac{dy}{y} = -7x\,dx$$

$$\int \frac{dy}{y} = \int -7x\,dx$$

$$\ln y = -\frac{7}{2}x^2 + c$$

$$e^{\ln y} = e^{-\frac{7}{2}x^2 + c}$$

$$y = C e^{-\frac{7}{2}x^2}$$

but $y(0) = 6$

$$6 = C e^{-\frac{7}{2}(0)^2}$$

$$6 = C e^0$$

$$C = 6$$

$$y = 6 e^{-\frac{7}{2}x^2}$$

2 $\frac{dy}{dt} = 450\ m/s$ $\qquad \frac{d^2y}{dt^2} = -2.5\ m/s^2$

$v = u + at$

$v = 0 \qquad u + at = 0$

$450 - 2.5t = 0$

$-2.5t = -450$

$2.5t = 450$

$t = 180\ s$

$$\int dy = \int_0^t u\,dt + \int_0^t at\,dt$$

$$y = ut + \frac{1}{2}at^2$$

$$= (450 \times 180) + \frac{1}{2}(-2.5)(180)^2$$

$$= 40.5\ \cancel{km}\ m$$

3 $\frac{dy}{dx} = 6x(y-1)^{\frac{2}{3}}$

$$\int \frac{dy}{(y-1)^{\frac{2}{3}}} = \int 6x\,dx$$

$$\int (y-1)^{-\frac{2}{3}}\,dy = 3x^2 + c$$

$$3(y-1)^{\frac{1}{3}} = 3x^2 + c$$

$$\sqrt[3]{y-1} = (x^2 + c)$$

$$(\sqrt[3]{y-1})^3 = (x^2 + c)^3$$

$$y - 1 = (x^2 + c)^3$$

$$y = (x^2 + c)^3 + 1$$

4 $N(t) = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$

$N(t) = \frac{N_0}{100}$ given $t_{\frac{1}{2}} = 5.\cancel{27}$ yrs

$\frac{N_0}{100} = N_0 \left(\frac{1}{2}\right)^{\frac{t}{5.27}}$

$\cancel{\ln}\ \ln\left(\frac{1}{100}\right) = \frac{t}{5.27}\left(\ln\left(\frac{1}{2}\right)\right)$

$-\ln(100) = \frac{-t}{5.27}\ln(2)$

$-2\ln 10 = \frac{-t}{5.27}\ln(2)$

$\cancel{t = 5.27}\ t = \frac{5.27 \times 2\ \ln 10}{\ln(2)}$ $= \cancel{\text{[illegible] yrs}} = 35.0131$ yrs

5 $\frac{dy}{dx} = 4x^3y - y$, $y(1) = -3$

$\frac{dy}{dx} = y(4x^3 - 1)$

$\int \frac{dy}{y} = \int (4x^3 - 1)\,dx$

$\ln y = x^4 - x + C$

$e^{\ln y} = e^{x^4 - x} \cdot e^{c}$

Let $e^c = A$

$y = e^{x^4 - x} \cdot A$

$-3 = e^{1^4 - 1} A$

$-3 = A$

$y = -3e^{x^4 - x}$

6 $y' = 1 + x + y + xy \qquad y(0) = 0$

$y' = 1 + x + y(1+x)$

$\frac{dy}{dx} - (1+x)y = 1+x$

$I.F = e^{\int -(1+x)dx} = e^{-x - \frac{x^2}{2}}$

$y \times I.F = \int e^{-x-\frac{x^2}{2}} (1+x)dx + c$

$-x - \frac{x^2}{2} = u$

$(-1-x)\,dx = du \Rightarrow (1+x)\,dx = -du$

$y \times I.F = \int e^u(-du) + c$

$y \times I.F = -e^u + c$

$y.I.F = -e^{-x-\frac{x^2}{2}} + c$

$y = -1 + Ce^{x + \frac{x^2}{2}}$

when $y(0) = 0$

$0 = -1 + Ce^{(0) + \frac{(0)^2}{2}}$

$C = 1$

$y = e^{x + \frac{x^2}{2}} - 1$

7 $(x^2+1)\frac{dy}{dx} + 3xy = 6x$

$$\frac{dy}{dx} + \frac{3x}{x^2+1}y = \frac{6x}{x^2+1}$$

$$I.F = e^{\int \frac{3x}{x^2+1}dx} = e^{\frac{3}{2}\int \frac{2x}{x^2+1}dx}$$

$$= e^{\frac{3}{2}\ln(x^2+1)}$$

$$= (x^2+1)^{3/2}$$

$$y(I(x)) = \int \frac{6x}{x^2+1} \cdot (x^2+1)^{3/2}\, dx$$

let $u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$y(I(x)) = \int 3 \cdot u^{1/2}\, dx \quad + c$$

$$y(I(x)) = \frac{3u^{3/2}}{3} \times 2 + c$$

$$= 2u^{3/2} + c$$

but $u = x^2+1$

$$y(I(x)) = 2 + c(x^2+1)^{-3/2}$$