ASSIGNMENT #2

Total Marks: 87 (To count 25% of the Final Grade)

Material covered in this Assignment: **Modules 3, 4 and 5**

Due Date: Thursday, June 23

IMPORTANT!

Please read the discussion forum posts on D2L under "Messages from your Course Supervisor" for details on how to submit your assignment.

All calculations must be shown to earn full marks.

It is expected that techniques from this class (*solving systems using matrices*) will be used for solving problems, when appropriate.

Marks will be deducted for not following these instructions.

1. Choose **three different non-zero integer** values for the empty boxes and find the determinant of the resulting matrix. [4 marks]

$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 3 & \square & 2 & 0 \\ -1 & 5 & \square & 4 \\ 0 & \square & -4 & 0 \end{pmatrix}$$

2. Let $A=\begin{pmatrix} -2 & 7 & 5 \\ 0 & 1 & 3 \\ -1 & 3 & 5 \end{pmatrix}$. Find the adjoint of A. [4 marks]

3. Use Cramer's rule to solve $\begin{aligned} 2x_1+3x_2-x_3&=7 \\ 3x_1-5x_2&=4 \\ x_1+x_3&=5 \end{aligned}$. [5 marks]

4. Prove that $det\ (A) = det\ (A^T)$ for a 2×2 matrix. [3 marks]

5. Consider the following four matrices:

$$A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix},\quad B=\begin{bmatrix} c & b & a \\ f & e & d \\ i & h & g \end{bmatrix},\quad C=\begin{bmatrix} 2a & 2b & 2c \\ d & e & f \\ g & h & i \end{bmatrix},\quad D=\begin{bmatrix} a-3c & b+5a & c \\ d-3f & e+5d & f \\ g-3i & h+5g & i \end{bmatrix}$$

If $det\ (A) = 3$, find the value of each of the following determinants.

a. $det\ (B)$ = ______ [4 marks]

b. $det\ (C)$ = ______

c. $det\ (D)$ = ______

d. $det\ (4AC^{-1})$ = ______

6. Let $A=\begin{pmatrix} x & -x & 3 \\ 0 & x+1 & 1 \\ x & -8 & x-1 \end{pmatrix}$. For what values of x does A^{-1} exist? [4 marks]

7. Let $V = \{(x,y) | x,y \in R\}$. Suppose addition and scalar multiplication are defined using the following **non-standard** rules where c is any real number.

$$(x_1, y_1) + (x_2, y_2) = (x_1 - x_2, 0)$$
$$c(x_1, y_1) = (-x_1, 5cy_1)$$

[4 marks]

a. Find the result of $(3,-2) + (-4,-3)$ under the above operations.

b. Find the result of $-3(2,-4)$ under the above operations.

c. Show that V, with respect to these operations of addition and scalar multiplication, is **not** a vector space by showing that **one** of the vector space axioms does not hold. Clearly identify the axiom you have chosen.

8. Let $V=\{(x,y)|x,y\in R\}$. Define addition on V as follows:

$$(x_1,y_1)+(x_2,y_2)=(x_1+2x_2-1,y_1+y_2+2)$$

Find the zero vector.

9. Let $V=\{(x,y)|x,y\in R\}$. Define addition on V as follows:

$$(x_1,y_1)+(x_2,y_2)=(x_1+2x_2-1,y_1+y_2+2)$$

Is addition commutative? Answer this question algebraically (***do not pick specific numerical values***).

10. a. Is the set of vectors $W=\{(x,y,z)|x,y,z\in R,x=z+2\}$ subspace of $\boldsymbol{R}^3$?

[5 marks]

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QUESTION #10 CONTINUED

b. Is the set of matrices of the form $\begin{pmatrix} p & t \\ -t & q \end{pmatrix}$ a subspace of $\boldsymbol{M}_{22}$?

11. For what value(s) of r (if any) is the vector (-2, 14, r) a linear combination of the vectors (2, 6, -1) and (4, 8, 3). [4 marks]

12. Consider the following sets of vectors in $\mathbb{R}^3$. **Explain** how you know that vectors in each set are linearly dependent. [2 marks]

Set A: { (2, 5, -1), (-1, -10, 2), (3, 9, 1), (1, 2, 0) }

Set B: { (4, -1, 9), (5, 1, 13), (1, 2, 4) }

13. Do the set of vectors S = { x^2+2x-4, $6x^2+7x+3$, $2x^2+x$ } a basis for P_2? Refer to the definition of a basis and show all your work for full marks. [5 marks]

14. a) Use the Gram Schmidt Orthogonalization Procedure to transform the basis { (0, 1, 2), (1, 1, 2), (1, 0, 1)) } into an orthogonal basis for $\boldsymbol{R}^3$.

[5 marks]

b) Use the dot product to verify your result from part (a).

15. Find a basis for the row space and the column space of the matrix $\begin{pmatrix} 1 & 3 & 2 & -5 \\ -2 & 3 & 6 & 7 \end{pmatrix}$. Write each basis in reduced echelon form. What is the rank of each basis? [4 marks]

16. Consider the **points** $A(-1,2,3)$ and $B(4,-1,2)$.

a. Determine the parametric and symmetric equations of the line that passes through the two given points. [4 marks]

b. Write the parametric equations of a **different** line that is parallel to the line above.

17. For what value(s) of k, if any, will the lines $\frac{x+1}{3}=\frac{y+2}{k}=z-3$ and $\vec{r}=(6,-1,0)+t(6,1-k,2)$ perpendicular? [3 marks]

18. Find the equation of a plane that passes through the points $A(3,-1,2)$, $B(6,4,1)$ and $C(2,0,-3)$. Does the point $(2,3,-9)$ lie on the plane?

[5 marks]

19. Find two unit vectors perpendicular to vector $\vec{u}$ = (7, 0, -1) and vector $\vec{v}$ = (1, -1, 1). [3 marks]

20. Let $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -2 & 2 \\ 3 & 0 & 1 \end{pmatrix}$.

a. Find the eigenvalues of A. [3 marks]

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QUESTION #20 CONTINUED

b. Find a basis and the dimension of each eigenspace. [5 marks]

21. The matrix $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$ is diagonalizable with eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Explain why A is diagonalizable, and find the diagonal matrix using the similarity transformation $C^{-1}AC = D$.

What were the original eigenvalues? [4 marks]

20. Are the matrices $A = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 3 \\ 0 & 2 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 3 & 4 \\ -1 & 4 & 0 \\ 2 & 3 & 4 \end{pmatrix}$ similar?

[3 marks]