

**Discrete Mathematics for Computer Science**

**Final Project**

**Create Portfolio Outlining the Solution to the following Questions**

**Weighting: 25%**

**Date Issued: March 6, 2023**

**Due Date: March 31, 2023**

**Functions**

**Question 1A**

Given that  $f(x) = 2x + 4$  and  $g(x) = 4x - 3$ :

- a.  $f^{-1}(x)$  [2 marks]
- b.  $fg(x)$  [2 marks]
- c. Represent  $f(x)$  on a graph [4 marks]

**Question 1B**

Let  $A = \{1, 2, 3, 4\}$  and  $B = \{p, q, r\}$ . Determine whether the relation  $R$  from  $A$  to  $B$  is a function. If it is a function, give its range. [2 marks each]

- a)  $R = \{(1, q), (2, r), (3, r), (4, q)\}$
- b)  $R = \{(1, p), (2, q), (3, r), (1, q), (4, p)\}$
- c)  $R = \{(2, r), (2, q), (3, p)\}$
- d)  $R = \{(1, q), (2, q), (3, q), (4, q)\}$

**Question 1C**

Let  $f$  be the function from  $X = \{0, 1, 2, 3, 4\}$  to  $Y$  defined by  $f(x) = 4x \bmod 5$ .

- i. Write  $f$  as a set of ordered pairs and draw a diagram to show the mapping.
- ii. Is  $f$  one-to-one?
- iii. Is  $f$  onto? [2 marks each]

**Prove the following cases by induction:**

**QUESTION 2A**

$$3+9+15+\dots+(6n-3) = 3n^2 \quad [5 \text{ MARKS}]$$

**QUESTION 2B**

$$1+5+9+\dots+(4n-3) = 2n^2 - n \quad [5 \text{ MARKS}]$$

**Arithmetic and Geometric Progression**

**QUESTION 3A**

For each of the following find the value of a, d and the nth term ( $u_n$  e.g.  $U_3$  – denotes the first three terms)

- a) 1, 4, 7, ....., ( $u_{10}$ )
- b) -8, -6, -4, ....., ( $u_{12}$ ) [2 marks each]

**Question 3B**

For each of the following arithmetic progression, find the values of a, d, and the  $S_n$  indicated

- a) 1, 3, 5, ....., ( $S_8$ )
- b) 10, 7, 4, ....., ( $S_{20}$ ) [3 marks each]

**Question 3C**

Find the term indicated for each of the geometric progressions.

- a) 1, 3, 9, . . . , ( $u_9$ )
- b) 4, -8, 16, . . . , ( $u_{10}$ ) [2 marks each]

**Question 3D**

Find the sum indicated for each of the following geometric series

- a)  $6 + 3 + \frac{3}{2} + \dots$  ( $S_{10}$ ) [3 marks]

### **Question 3E**

- a. Find the sum of the 5<sup>th</sup> terms of the geometric sequence whose first term is 54 and fourth term is 2.
- b. Find the sum of the 5<sup>th</sup> terms of an arithmetic sequence whose fourth and fifth terms are 13 and 15.

[3 marks each]

### **Question 4 -Permutation and Combination**

1. If a Bike's license plate comprises of three letter followed by four digits (0 to 9). How many Bike license plates can be generated?
2. How many seven-character passwords can be generated if the first character must be a letter (A to Z) and the other 6 characters may be a digit or a letter of the alphabet?
3. If the Lakers Basketball team has applications from 7 Universities for hosting its intercollegiate Basketball championships in 2022 and 2023, how many ways may they select the hosts for these games championships:
  - a) if they are **not** both to be held at the same University?
  - b) if they may both be held at the same University?
4. If there are 30 students in a class, how many ways can a **monitor** and a **student counsellor** be chosen.
  - a. If the same person may be both student counsellor and monitor?
  - b. If the same person may not be both student counsellor and monitor?
5. How many four-letter words can be made from the letters: MOUSE?
6. How many three-letter words can be made from the letters: KINGSTON?
7. In how many ways may 8 people form a circle for a folk dance?
8. In a general election, there are five candidates for mayor, five candidates for city treasurer, and two candidates for county attorney. In how many ways may voters mark their ballots
  - a) if they vote in all three of the races?
  - b) if they exercise their right not to vote in any or all of the races?

9. A multiple-choice test consists of 20 questions, each permitting a choice of 4 alternatives. In how many ways may a student fill in the answers if they answer each question?
10. In how many ways may one A, three B's, two C's, and one F be distributed among seven students in a Discrete Mathematics class?

[10 marks]

### **Probability**

#### **Question 5A**

Suppose that a die is loaded so that the numbers 1 to 4 are equally likely, the number 5 is two times as likely as the number 6 and the number 6 is three times as likely as two.

Calculate the probability of 2, 5 and 6.

[6 marks]

#### **Question 5B**

The numerals 4, 5, 6 and 7 are printed on identical cards which are placed in a box. One of the cards is drawn at random, the numeral is recorded and the card is placed back in the box. A second card is drawn and the numeral is recorded.

Write the set of ordered pairs which is the sample space for the experiment.

The two numerals recorded are summed and the possible values of the sum are a discrete random variable. Complete the table below to show all the possible values of the discrete random variables  $X$  and associated probabilities  $P(X)$

Calculate the expectation,  $E(X)$  of the discrete random variables.

[7 marks]

#### **Question 5C**

Peter has ten coloured cubes in a bag. Three of the cubes are red and 7 are blue. He removes a cube at random from the bag and notes the colour before replacing it. He then chooses a second cube at random. Record the information in a tree diagram.

[5 marks]

#### **Question 5D**

Peter has ten coloured cubes in a bag. Three of the cubes are red and seven are blue. He removes a cube at random from the bag and notes the colour but does not replace it. He then chooses a second cube at random. Record the information in a tree diagram.

[5 marks]

**Question 6**

1. Describe three useful applications of graphing algorithms in computer science. [3 marks]
2. What is an Euler cycle? [2 marks]
3. Construct a Direct Graph for the Mico University Campus road network. [5 marks]
4. Does the graph contain an Euler Cycle? Give a reason for your answer. [2 marks]

**[TOTAL 100 MARKS]**