

$$(3) (x-5)^2(x+2) < 0$$

$$\text{soln: } [\text{Interval Notation } (-\infty, -2)]$$

Signs for the factors.

	$x < -2$	$x = -2$	$-2 < x < 5$	$x = 5$
$(x-5)^2$	+	+	+	0
$x+2$	-	0	+	+
$(x-5)^2(x+2)$	-	0	+	0

The interval that satisfy required condition: $< 0 \Rightarrow \underline{x < -2}$

$$(b) x^3 - 4x^2 > 0$$

soln. applying exponent rule. $a^{b+c} = a^b a^c$

$$x^3 = x x^2$$

$$= x x^2 - 4x^2$$

factor common term $x^2 \Rightarrow x^2(x-4)$

$$x^2(x-4) > 0$$

find signs of x^2 and $x-4$

	$x < 0$	$x = 0$	$0 < x < 4$	$x = 4$
x^2	+	0	+	+
$x-4$	-	-	-	0
$x^2(x-4)$	-	0	-	0

The interval soln) $x > 4$

[Interval notation $(4, \infty)$]

The favourable interval = $x > 4$

(c) $2x^3 > -8x^2$

Soln. $2x^3 + 8x^2 > -8x^2 + 8x^2$

$\Rightarrow (2x^3 + 8x^2 > 0) \cdot \frac{1}{2}$

divide both sides by 2. $\Rightarrow x^3 + 4x^2 > 0$

factor: $x^3 + 4x^2 = x^2(x+4)$

$x^2(x+4) > 0$

	$x < -4$	$x = -4$	$-4 < x < 0$	$x = 0$
x^2	+	+	+	0
$x+4$	-	0	+	+
$x^2(x+4)$	-	0	+	0

Therefore the intervals that satisfy the required condition $-4 < x < 0$ or $x > 0$

Soln. $\left[-4 < x < 0 \text{ or } x > 0 \right]$
Interval notation: $(-4, 0) \cup (0, \infty)$

d. $(x-1)(x-2)(x-3) \leq 0$

in this case we find signs of the factors

	$x < 1$	$x = 1$	$1 < x < 2$
$x-1$	-	0	+
$x-2$	-	-	-
$x-3$	-	-	-
$(x-1)(x-2)(x-3)$	-	0	+

Ans:

Intervals that satisfy the eqn: $x \leq 0$.
 $x \leq 1$ or $x = 1$ or $x = 2$ or $2 < x < 3$ or $x = 3$

(e) $x^3 - 2x^2 - 3x > 0$

factor $x^3 - 2x^2 - 3x = x(x+1)(x-3)$

Common term:

$x(x^2 - 2x - 3)$

$\Rightarrow (x^2 + x) + (-3x - 3)$

factor common term:

$x^2 + x \rightarrow x(x+1)$

on second part $-3x - 3 \Rightarrow -3(x+1)$

$\Rightarrow (x+1)(x-3)x$

$\therefore x(x+1)(x-3) > 0$

Defind Signs:

	$x < -1$	$x = -1$	$-1 < x < 0$	$x = 0$
x	-	-	-	0
$x+1$	-	0	+	+
$x-3$	-	-	-	-
$x(x+1)(x-3)$	-	0	+	0

$\Rightarrow$ The interval that satisfy required condition
 $x > 0$ $-1 < x < 0$ or $x > 3$

(f) $x^4 > x^2$

Subtract x^2 from both sides

$$x^4 - x^2 > x^2 - x^2$$

$$x^4 - x^2 > 0$$

factor

$$x^2(x^2 - 1)$$

factor $(x^2 - 1)$

$$= (x+1)(x-1)$$

$$\therefore x^2(x+1)(x-1)$$

we find signs of $x^2(x+1)(x-1)$

	$x < -1$	$x = -1$	$-1 < x < 0$	$x = 0$
x^2	+	+	+	0
$x+1$	-	0	+	+
$x-1$	-	-	-	-
$x^2(x+1)(x-1)$	+	0	-	0

The required intervals ~~are~~ > 0

$$x < -1 \text{ or } x > 1$$

(g) $x^4 > 1^4$

for $u^n > a^n$ if n is even then $u > \sqrt[n]{a}$ or $u < -\sqrt[n]{a}$

$$x < -1^4 \text{ or } x > 1^4$$

$$1^4 = 1$$

using $1^4 = 1$

$$\Rightarrow \underline{x < -1 \text{ or } x > 1}$$

(h) $\frac{x+1}{x-1} > 0$

we find the signs of the factors

$$\frac{(x+1)}{(x-1)}$$

find zeros of the denominator
 $x-1: x=1$

	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$
$x+1$	-	0	+	+
$x-1$	-	-	-	0
$\frac{x+1}{x-1}$	+	0	-	undefined

$\therefore$ The intervals that satisfy required condition

$$: > 0 \Rightarrow x < -1 \text{ or } x > 1$$

(1) $\frac{(x-1)(x+1)}{x} \leq 0$

We find the signs of the factors.

	$x < -1$	$x = -1$	$-1 < x < 0$	$x = 0$
$x-1$	-	-	-	-
$x+1$	-	0	+	+
x	-	-	-	0
$\frac{(x-1)(x+1)}{x}$	-	0	+	undefined

$\therefore$ the required interval that satisfy required conditions

$= x < -1$ or $x = -1$ or $0 < x < 1$ or $x = 1$

(3) $\frac{(x-2)^2}{x^2-1}$

factor $x^2-1 = (x+1)(x-1)$

$= \frac{(x-2)^2}{(x+1)(x-1)}$

$\frac{(x-2)^2}{(x+1)(x-1)} \geq 0$

	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$
$(x-2)^2$	+	+	+	+
$x+1$	-	0	+	+
$x-1$	-	-	-	0
$\frac{(x-2)^2}{(x+1)(x-1)}$	+	undefined	-	undefined

the required interval that satisfy ≥ 0

we get the signs of the factors $= x < -1$ or $-1 < x < 2$ or $x = 2$ or $x > 2$

(4) $\frac{x+4}{x-2} \leq 1$

Subtract 1 from both sides

$\frac{x+4}{x-2} - 1 \leq 1 - 1$

convert element into fraction $1 = \frac{1(x-2)}{x-2}$

$= \frac{x+4}{x-2} - \frac{1 \cdot (x-2)}{x-2}$

$= \frac{x+4 - 1(x-2)}{x-2}$

$\frac{x+4-x+2}{x-2}$

$\Rightarrow \frac{x-x+4+2}{x-2} = \frac{6}{x-2}$ we take sides

by 6 $= \frac{6}{x-2} \leq \frac{0}{6} \Rightarrow \frac{1}{x-2} \leq 0$

find signs of $x-2$

	$x < 2$	$x = 2$	$x > 2$
$x-2$	-	0	+
$\frac{1}{x-2}$	-	undefined	+

the interval that satisfy the required condition: ≤ 0

$\Rightarrow x < 2$

$$(l) \frac{3x-5}{x+2} \leq 2$$

Subtract 2 from both sides

$$\frac{3x-5}{x+2} - 2 \leq 2 - 2$$

Convert 2 into fraction

$$\frac{3x-5}{x+2} - \frac{2(x+2)}{x+2}$$

$$= \frac{3x-5-2x-4}{x+2} = \frac{x-9}{x+2}$$

We find the signs of factors $\frac{(x-9)}{(x+2)}$

	$x < -2$	$x = -2$	$-2 < x < 9$	$x = 9$
$x-9$	-	-	-	0
$x+2$	-	0	+	+
$\frac{(x-9)}{(x+2)}$	+	undefined	-	0

The required interval to satisfy required

Condition: ≤ 0

$$-2 < x < 9 \text{ or } x = 9$$

$$(m) \frac{1}{x-2} < \frac{2}{3x-9}$$

We subtract $\frac{2}{3x-9}$ from both sides

$$\frac{1}{x-2} - \frac{2}{3x-9} < \frac{2}{3x-9} - \frac{2}{3x-9}$$

$$\frac{1}{x-2} - \frac{2}{3x-9} < 0$$

$$\frac{1}{x-2} - \frac{2}{3(x-3)}$$

$$= \frac{3(x-3) - 2(x-2)}{3(x-3)(x-2)}$$

$$= \frac{3x-9-2x+4}{3(x-3)(x-2)}$$

$$= \frac{x-5}{3(x-3)(x-2)} < 0$$

Multiply both sides by 3

$$\Rightarrow \frac{x-5}{(x-3)(x-2)} < 0$$

$$\frac{x-5}{(x-3)(x-2)} < 0$$

get the signs of factors

	$x < 2$	$x = 2$	$2 < x < 3$	$x = 3$
$x-5$	-	-	-	-
$x-2$	-	0	+	+
$x-3$	-	-	-	0
$\frac{x-5}{(x-3)(x-2)}$	-	undefined	+	undefined

The required interval to satisfy the required

Condition: < 0

$$x < 2 \text{ or } 3 < x < 5$$

$$(i) \frac{(3-x)^2(2x+1)}{x^3-1} < 0$$

factor x^3-1 :

$$= \frac{(-x+3)(2x+1)}{(x-1)(x^2+x+1)}$$

$$\frac{(3-x)^3(2x+1)}{(x-1)(x^2+x+1)} < 0$$

Then factor $(3-x)^3$

$$= \frac{(-(x-3))(2x+1)}{(x-1)(x^2+x+1)} < 0$$

$$(ii) \frac{x^2(3+x)(x+4)}{(x+5)(x-1)} \geq 0$$

Soln:

$$x^2 \geq 0: x = 0$$

$$x^2 > 0: x < 0 \text{ or } x > 0$$

$$3+x = 0 \quad x = -3$$

$$3+x < 0 \quad x < -3$$

$$3+x > 0 \quad x > -3$$

$$x-3 = 0 \Rightarrow x-3=0 \quad x=3$$

$$x-3 < 0 \Rightarrow x < 3$$

$$x-3 > 0 \Rightarrow x > 3$$

$$2x+1=0: x = -\frac{1}{2}$$

$$2x+1 < 0: x < -\frac{1}{2}$$

$$2x+1 > 0: x > -\frac{1}{2}$$

$$x-1$$

$$x-1=0 \quad x=1$$

$$x-1 < 0: x < 1$$

$$x-1 > 0: x > 1$$

$$x^2+x+1 > 0$$

$$(x-1)(x^2+x+1): x=1$$

$$\text{Soln set } \left\{ \frac{1}{2} < x < 1 \text{ or } x > 3 \right\}$$

$$x+4=0: x=-4$$

$$x+4 < 0: x < -4$$

$$x+4 > 0: x > -4$$

$$x+5=0 \quad x=-5$$

$$x+5 < 0 \quad x < -5$$

$$x+5 > 0 \quad x > -5$$

$$x-1=0 \quad x=1$$

$$x-1 < 0 \quad x < 1$$

$$x-1 > 0 \quad x > 1$$

(1) Continued. for denominator

$$x+5=0 \quad x=-5$$

$$x+5 < 0 \quad x < -5$$

$$x+5 > 0 \quad x > -5$$

$$x-1=0 \quad x=1$$

$$x-1 < 0 \quad x < 1$$

$$x-1 > 0 \quad x > 1$$

Solution.
The required interval

$$= x(x < -5 \text{ or } -4 \leq x \leq -3 \text{ or } x > 1)$$

Question 11

$$f(x) = x^4 - 1 \quad g(x) = -2x^2 + 2$$

$$f(x) \leq g(x)$$

$$x^4 - 1 = 2x^2 + 2$$

$$x^4 - 2x^2 - 3 = 0 \quad \text{let } x^2 = k$$

$$k^2 - 2k - 3 = 0$$

$$k^2 - 3k + k - 3 = 0$$

$$k(k-3) + (k-3) = 0$$

$$k=3, k=-1$$

$$x^2 = 3 \quad x = \sqrt{3}$$

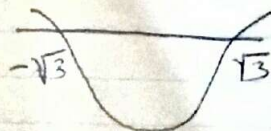
$$x^2 = -1 \quad x = -\sqrt{3}$$

now

$$x^4 - 1 \leq 2x^2 + 2$$

$$x^4 - 2x^2 - 3 \leq 0$$

$$(x^2-3)(x^2+1) \leq 0 \quad [-\sqrt{3}, \sqrt{3}]$$



Question 5.

$$f(x) = x^4 - 4$$

$$g(x) = 3x^2$$

Solve $f(x) \leq g(x)$

Soln:

$$x^4 - 4 \leq 3x^2$$

$$x^4 - 4 - 3x^2 \leq 0$$

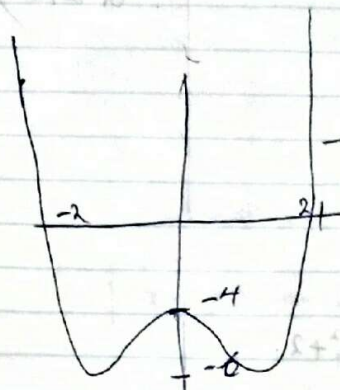
$$x^4 - 3x^2 - 4 \leq 0$$

$$(x^2 - 4)(x + 1)^2 \leq 0$$

$$(x^2 - 4) \leq 0 \quad (x + 1)^2 \leq 0$$

$$x = 2, -2 \quad x = -1, -1$$

$$x = -2, 2, -1, -1$$

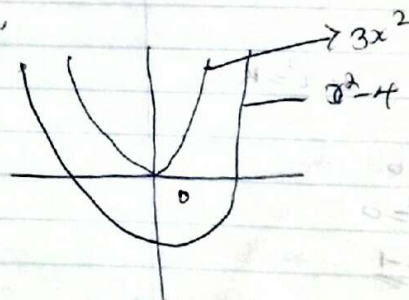


$$\rightarrow x^4 - 3x^2 - 4 \leq 0$$

The graph of f lies below ' g '

$$f(x) \geq 0 \Rightarrow x^2 \geq 4$$

$$\{x: -2 \leq x \leq 2\}$$



Question 6.

$$\frac{80x + 5000}{x} \leq 100$$

$$80x + 5000 \leq 100x$$

$$5000 \leq 20x$$

$$x \geq 250$$

Therefore at least 250 bicycles must be produced each day for average cost to be no more than \$100.