

Part I:

- ① Line passing through the point $(2, 4)$ and parallel to $5x + 6y = 7$

⇒ Since we want a parallel line, then we retain the first two terms and plug the point.

$$5x + 6y = 5(2) + 6(4) = 10 + 24 = 34$$

Answer: $5x + 6y = 34$

- ② Line passing through $(-1, 3)$ and perpendicular to $11x + 2y = -13$

⇒ Note that the slope of the line we want must be negative reciprocal of our reference line.

$$11x + 2y = -13 \Rightarrow y = -\frac{11}{2}x - \frac{13}{2}$$

$$\text{slope} = -\frac{11}{2} ; \text{negative reciprocal} = \frac{2}{11}$$

We use $\frac{2}{11}$ as our slope. Hence,

$$y = \frac{2}{11}x + b ; \text{ we then plug-in the required point to find } b$$

$$3 = \frac{2}{11}(-1) + b$$

$$33 = -2 + 11b$$

$$b = \frac{35}{11}$$

$$\text{hence, } y = \frac{2}{11}x + \frac{35}{11}$$

reducing to standard form,

$$2x - 11y = -35$$

③ Line passing through points $(-7, 9)$ and $(3, 2)$

⇒ Using the slope formula, the slope of the line is :

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 2}{-7 - 3} \text{ or } \frac{2 - 9}{3 - (-7)} = -\frac{7}{10}$$

then, we can use the point-slope form of the line to determine the equation.

$(y - y_0) = m(x - x_0)$; we can use either given points

$$\rightarrow y - 9 = -\frac{7}{10}(x + 7)$$

$$10y - 90 = -7x - 49$$

$$7x + 10y = 90 - 49 = 41$$

Answer: $7x + 10y = 41$

④ Line passing through $(-8, 3)$ and $(-8, -3)$

notice that the x -value does not change.

therefore, we expect an equation that is independent of y -values. the most

precise answer is $x = -8$.

we can verify this in a longer method :
using the point-slope form,

$$y - y_0 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_0)$$

$$\left(\frac{x_2 - x_1}{y_2 - y_1} \right) (y - y_0) = x - x_0$$

$$\begin{aligned} \frac{-8 + 8}{3 - (-3)} (y - 3) &= x + 8 \\ 0 &= x + 8 \\ -8 &= x \end{aligned}$$

hence, answer is
 $x = -8$

II. Given $f(x) = x+2$ and $g(x) = \frac{-7}{x-6}$

⑤ Find the x 's for which $f(x) = g(x)$

⇒ we equate the two functions straightforwardly,

$$2+x = \frac{-7}{x-6}$$

$$(x-6)(x+2) = -7$$

$$x^2 - 6x + 2x - 12 = -7$$

$$x^2 - 4x - 5 = 0$$

we factor the quadratic equation,

$$x^2 - 5x + x - 5 = 0$$

$$x(x-5) + (x-5) = 0$$

$$(x+1)(x-5) = 0$$

$$x = 5 \text{ and } -1$$

Answer:

$$x = 5 \text{ } \{ -1$$

you can also use the quadratic formula and yield the same answers

⑥ Find $g[f(x)]$.

⇒ We are required to find $g(x)$ as a composition by $f(x)$. This simply means that $g[x+2]$ is asked and we treat " $x+2$ " as the value of $g(x)$. By plugging $f(x) = x+2$,

$$g[f(x)] = \frac{-7}{f(x)-6} = \frac{-7}{(x+2)-6} = \frac{-7}{x-4}$$

$$\text{Answer: } g[f(x)] = \frac{-7}{x-4}$$

III ⑦ Find the average rate of change of $f(x) = 2x^2 - 5$ over the x -interval $[-1, 5]$.

⇒ to find the average rate of change or slope in the interval, we must first find the endpoints of this interval.

$$f(-1) = 2(-1)^2 - 5 = 2 - 5 = -3 \quad ; \quad P_1(-1, -3)$$

$$f(5) = 2(5)^2 - 5 = 50 - 5 = 45 \quad ; \quad P_2(5, 45)$$

using the slope formula, we find the slope between these points

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45 - (-3)}{5 - (-1)} = \frac{48}{6} = 8$$

Answer: the average rate of change is 8.

IV.

$$\textcircled{8} \quad f(x) = \frac{1}{2}(x+7)^2 = \frac{1}{2}(x^2 + 14x + 49) = \frac{1}{2}(x+7)^2 + 0$$

vertex: $(-7, 0)$

- hence, neither up nor down by any quantity
- vertex at $x = -7$, graph is shifted 7 units to the left
- a factor of $\frac{1}{2}$ indicates a horizontal stretch causing a vertical shrink
- since the x^2 term is positive, it is oriented right side up.

$$⑨ f(x) = -3|x-2| + 5$$

$$= \begin{cases} -3(x-2) + 5 & \text{for } x-2 \geq 0 \\ -3(2-x) + 5 & \text{for } x-2 < 0 \end{cases}$$

vertex: (2, 5)

- graph is up by 5 units
- vertex at $x=2$, graph is shifted 2 units to the right
- a factor of -3 indicates horizontal shrink causing a vertical stretch
- first term is negative, graph is oriented upside down.

Ⅴ.

$$⑩ \text{ Find the inverse of } f(x) = \frac{2x-1}{3x+3}$$

for an easier notation, $y = \frac{2x-1}{3x+3}$

to find the inverse function, switch x & y variables such that:

$$x = \frac{2y-1}{3y+3}$$

solving for y ,

$$x(3y+3) = 2y-1$$

$$3xy + 3x = 2y - 1$$

$$3x + 1 = 2y - 3xy$$

$$3x + 1 = y(2 - 3x)$$

$$\frac{3x+1}{2-3x} = y$$

Answer:

$$f^{-1}(x) = \frac{3x+1}{2-3x}$$

VII.

⑪ Domain: $[-8, +\infty)$

- the graph begins at $x = -8$ and extends infinitely to the right

Range: $(-\infty, 8]$

- since the max. value of the graph is at $y = 8$ in the $+y$ - direction and the graph extends downwards infinitely

Increasing: $[-5, -1] \cup [3, 6]$

- take note that this is in interval notation; these are the intervals where the average slope is positive; closed interval since endpoints are closed

Decreasing: $[-8, -5] \cup [6, +\infty)$

- negative slope at the interval $[-8, -5]$ and closed since points are part of the graph; negative slope at $[6, +\infty)$.

constant: $[-1, 3]$

- a flat line indicates a constant graph; closed interval since endpoints are part of the graph.

$F(x) > 0 : [-8, -6) \cup (-3, 9)$

$F(x) < 0 : (-6, -3) \cup (9, +\infty)$

Zeros: $x = -6, -3, 9$

- these are simply the x -intercepts

y -intercept: $y = 3$

- set $x = 0$ or look at the point at w/c $x = 0$

VII given the piecewise function:

$$f(x) = \begin{cases} x + 5 & \text{for } x \leq 2 \\ -x + 3 & \text{for } x > 2 \end{cases}$$

⑫ Evaluate $f(3)$

$\Rightarrow$ since $x = 3$ is in the interval $x > 2$, then:

$$f(3) = -(3) + 3 = -3 + 3 = 0$$

$$\boxed{f(3) = 0}$$

⑬ Evaluate $f(-2)$

$\Rightarrow$ since $x = -2$ is in the interval $x \leq 2$, then:

$$f(-2) = (-2) + 5 = -2 + 5 = 3$$

$$\boxed{f(-2) = 3}$$

⑭ Evaluate $f(2)$

$\Rightarrow$ since $f(2)$ is included in the interval $x \leq 2$, then:

$$f(2) = (2) + 5 = 7$$

$$\boxed{f(2) = 7}$$

VIII. Given: $f(x) = x^2 - x - 10$

$$g(x) = 2x - 7$$

$$k(x) = \frac{4x - 8}{3 - 2x}$$

⑮ $(k \circ g)(x)$, simplify

$$(k \circ g)(x) = k[g(x)]$$

$$= \frac{4(g(x)) - 8}{3 - 2(g(x))}$$

$$\begin{aligned} & \frac{4(2x - 7) - 8}{3 - 2(2x - 7)} \\ &= \frac{8x - 28 - 8}{3 - 4x + 14} = \frac{8x - 36}{-4x + 17} \end{aligned}$$

$$\boxed{\text{Answer: } (k \circ g)(x) = \frac{8x - 36}{-4x + 17}}$$

⑥ $f[g(5)]$ evaluate

⇒ let us first find $g(5)$.

$$g(5) = 2(5) - 7 = 10 - 7 = 3$$

$$f[g(5)] = f(3) = (3)^2 - (3) - 10$$

$$f(3) = 9 - 3 - 10 = 9 - 13 = -4$$

$$f(3) = -4$$

$$\boxed{\text{Answer: } f(3) = -4}$$

⑦ $f(2x-1)$ simplify

$$\begin{aligned} f(2x-1) &= (2x-1)^2 - (2x-1) - 10 \\ &= 4x^2 - 4x + 1 - 2x + 1 - 10 \end{aligned}$$

$$f(2x-1) = 4x^2 - 6x - 8$$

$$\boxed{\text{Answer: } f(2x-1) = 4x^2 - 6x - 8}$$