

Solution 10

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{2x} =$$

Taking log and antilog

$$\Rightarrow \lim_{x \rightarrow \infty} e^{\log \left(1 - \frac{1}{x}\right)^{2x}} = \lim_{x \rightarrow \infty} e^{2x \cdot \log \left(1 - \frac{1}{x}\right)}$$

$$\Rightarrow \lim_{x \rightarrow \infty} e^{2x \left[\left(-\frac{1}{x}\right) + \frac{\left(-\frac{1}{x}\right)^2}{2} + \frac{\left(-\frac{1}{x}\right)^3}{3} + \dots \infty \right]} \left[\log(1+x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \right]$$

$$\Rightarrow \lim_{x \rightarrow \infty} e^{2 \left[-1 + \frac{1}{2x} - \frac{1}{3x^2} + \dots \infty \right]}$$

$$\Rightarrow \boxed{e^{-2}}$$

Solution 11

$$a_n = \frac{6n^3 + 7}{n^3}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{6n^3 + 7}{n^3} = \lim_{n \rightarrow \infty} \left(6 + \frac{7}{n^3} \right)$$

$$= \textcircled{6}$$

Solution 12

I would limit compare to $\sum \frac{1}{\sqrt{n}}$

I come up with this by looking at dominant terms in the numerator and denominator of the n^{th} term of the given series:

numerator : denominator term is n

We know that $\sum \frac{1}{\sqrt{n}}$ is a divergent p -series with

$p = \frac{1}{2}$ which is less than 1.

$$\sum_{n=1}^{\infty} \frac{4n}{\sqrt{n^3+5}}$$

$$\lim_{n \rightarrow \infty} \frac{4n}{\sqrt{n^3+5}} = \lim_{n \rightarrow \infty} \frac{4n^{3/2}}{n^{3/2}+5}$$
$$\frac{1}{\frac{1}{\sqrt{n}}}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{4}{1 + \frac{5}{n^{3/2}}} = 4$$

so, it is divergent.

Solution 13

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

Explanation

$$\frac{d \ln x}{dx} = \frac{1}{x}$$

so

$$\frac{d \ln(\ln x)}{dx} = \frac{1}{\ln x} \cdot \frac{d \ln x}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$= \frac{1}{x \cdot \ln x}$$

so

$$\int \frac{1}{x \ln x} dx = \ln(\ln x) + C$$

Then

$$\lim_{x \rightarrow \infty} \ln(\ln x) = \infty$$

so by the integral test :

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n} \text{ diverges.}$$

Solⁿ 14.

$\sum_{n=1}^{\infty} \frac{(-1)^n}{3n-1}$ is absolutely convergent if $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{3n-1} \right|$

i.e. $\sum_{n=1}^{\infty} \frac{1}{3n-1}$ is convergent.

~~Using Ratio test,~~

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{1}{3(n+1)-1} = \lim_{n \rightarrow \infty} \frac{1}{3n+2}$$

$\Rightarrow$ Using Limit Comparison test, $a_n = \frac{1}{3n-1}$ & $b_n = \frac{1}{n}$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{3n-1}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{3 - \frac{1}{n}}$$

$$= \frac{1}{3} \neq 0 \neq \infty$$

As, $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent, so $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n-1}$ is not

absolutely convergent.

Sol.ⁿ 15

$$\sum_{n=1}^{\infty} \frac{1}{n 4^n} (x-3)^n$$

Using Ratio test,

$$R = \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x-3)^n (n+1) 4^{n+1}}{(x-3)^{n+1} n \cdot 4^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{4^n \left(1 + \frac{1}{n}\right)}{(x-3)^n} \right| = \lim_{n \rightarrow \infty} \left| \left(\frac{4}{x-3}\right)^n \left(1 + \frac{1}{n}\right) \right|$$

If $(x-3) > 4$, $R = 0$

If $(x-3) < 4$, $R = \infty$

If $x-3 = 4$, $R = 1$

Sol.ⁿ 16

a) $x^2 + y^2 = 37$

$x = r \cos \theta$ $y = r \sin \theta$ (in polar form)

$$\Rightarrow (r \cos \theta)^2 + (r \sin \theta)^2 = 37 (r \sin \theta)$$

$$\Rightarrow r^2 \cos^2 \theta + r^2 \sin^2 \theta = 37 r \sin \theta$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 37 r \sin \theta$$

$$\Rightarrow \frac{r^2}{r} = 37 \sin \theta$$

$$\Rightarrow \boxed{r = 37 \sin \theta}$$

16 b)

$$r = 5 \cos \theta$$

multiply by r both side

$$\Rightarrow r^2 = 5r \cos \theta$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 5r \cos \theta \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow r^2 \cos^2 \theta + r^2 \sin^2 \theta = 5r \cos \theta$$

$$\Rightarrow (r \cos \theta)^2 + (r \sin \theta)^2 = 5r \cos \theta$$

$$\boxed{x^2 + y^2 = 5x}$$

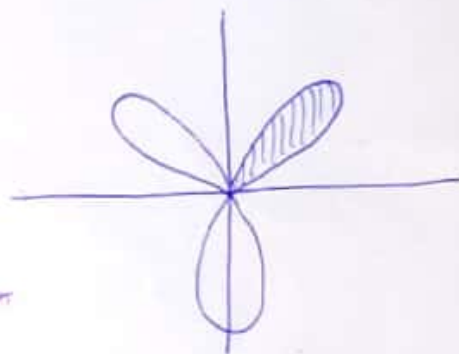
Soln 17

$$r = 2 \sin 3\theta$$

$$\text{when } r = 0 \Rightarrow \sin 3\theta = 0$$

$$\Rightarrow 3\theta = n\pi$$

$$\Rightarrow \theta = \frac{n\pi}{3} = \pm \frac{\pi}{3}, \pm \frac{2\pi}{3}, \pm \pi$$



let's find out the shaded petal,

$$A = \frac{1}{2} \int_{\pi/3}^{2\pi/3} r^2(\theta) d\theta = \frac{1}{2} \int_{\pi/3}^{2\pi/3} (2 \sin 3\theta)^2 d\theta$$

$$= \frac{1}{2} \int_{\pi/3}^{2\pi/3} 4 \sin^2 3\theta d\theta = 2 \int_{\pi/3}^{2\pi/3} \left(\frac{1 - \cos 6\theta}{2} \right) d\theta$$

$$= \left[\theta - \frac{\sin 6\theta}{6} \right]_{\pi/3}^{2\pi/3} = \left[\frac{2\pi}{3} - \frac{\pi}{3} - \frac{\sin 4\pi}{6} + \frac{\sin 2\pi}{6} \right]$$

$$= \frac{\pi}{3}$$

sol-18

a)

fourth Taylor polynomial for f , centered at 1 is

$$\frac{f'''(1)(x-1)^3}{3!} = \frac{2}{(1)^3} \frac{(x-1)^3}{3!} = \frac{2(x-1)^3}{3!} = \frac{(x-1)^3}{3}$$

b)

$$f(x) = \ln x$$

$$f(1) = 0$$

$$f'(x) = \frac{1}{x}$$

$$f'(1) = 1$$

$$f''(x) = -\frac{1}{x^2}$$

$$f''(1) = -1$$

$$f'''(x) = \frac{(-1)^2 \cdot (2)}{x^3}$$

$$f'''(1) = 2$$

$$f^{(n)}(x) = \frac{(-1)^n (n-1)!}{x^n}$$

$$f^{(n)}(1) = (-1)^n (n-1)!$$

Taylor series centered at 1 is

$$\sum_{n=1}^{\infty} \frac{(x-1)^n (-1)^n}{n}$$