

$$\int_1^4 (x^2 - 4x + 2) \, dx$$

The limit definition of an integral is:

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$$

where

$$\Delta x = \frac{b-a}{n}$$

$$x_i^* = a + \left(\frac{b-a}{n}\right)i$$

Using the above:

$$\Delta x = \frac{4-1}{n} = \frac{3}{n}$$

$$x_i^* = 1 + \frac{3i}{n}$$

$$\int_1^4 (x^2 - 4x + 2) \, dx$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(1 + \frac{3i}{n}\right)^2 - 4\left(1 + \frac{3i}{n}\right) + 2 \right] \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(1 + \frac{6i}{n} + \frac{9i^2}{n^2}\right) - \left(4 + \frac{12i}{n}\right) + 2 \right] \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{9i^2}{n^2} - \frac{6i}{n} - 1 \right) \cdot \frac{3}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{27i^2}{n^3} - \frac{18i}{n^2} - \frac{3}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \left[\sum_{i=1}^n \left(\frac{27i^2}{n^3} \right) - \sum_{i=1}^n \left(\frac{18i}{n^2} \right) - \sum_{i=1}^n \left(\frac{3}{n} \right) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{27}{n^3} \sum_{i=1}^n i^2 - \frac{18}{n^2} \sum_{i=1}^n i - \frac{3}{n} \sum_{i=1}^n 1 \right]$$

Using

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n 1 = n$$

$$= \lim_{n \rightarrow \infty} \left[\frac{27}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right) - \frac{18}{n^2} \left(\frac{n(n+1)}{2} \right) - \frac{3}{n} (n) \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{9}{n^2} \left(\frac{(n+1)(2n+1)}{2} \right) - \frac{9}{n} (n+1) - 3 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{2n^2} [3(n+1)(2n+1) - 6n(n+1) - 2n^2]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{2n^2} [(6n^2 + 9n + 3) - (6n^2 + 6n) - 2n^2]$$

$$= \lim_{n \rightarrow \infty} \frac{3}{2n^2} [-2n^2 + 3n + 3]$$

$$= \lim_{n \rightarrow \infty} (-3) + \lim_{n \rightarrow \infty} \left(\frac{9}{2n} \right) + \lim_{n \rightarrow \infty} \left(\frac{9}{2n^2} \right)$$

$$= -3 + 0 + 0$$

$$= -3$$