

Quiz 4

Show all relevant steps for each question.

1. For the function $f(x) = 9x^3 - 4x$, please find all of the following and then use that information to sketch the curve. **(Steps may be out of order)**

- a) All x and y intercepts.

$$\underline{x\text{-int}} \quad f(x) = 0 \Rightarrow 9x^3 - 4x = 0$$

$$x(9x^2 - 4) = 0$$

$$x(3x+2)(3x-2) = 0$$

$$(0, 0), \left(-\frac{2}{3}, 0\right), \left(\frac{2}{3}, 0\right)$$

$$\underline{y\text{-int}} \quad f(0) = 0 \quad (0, 0)$$

- b) Whether it is symmetric to the x-axis, y-axis or the origin

$$f(-x) = -9x^3 + 4x = -f(x)$$

Symmetric to the origin.

- c) Behavior of the function as x becomes large

(1 Mark)

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

rises to the right and falls to the left.

- d) Domain and Range

(1 Mark)

$$D: (-\infty, \infty) \quad R: (-\infty, \infty)$$

e) Both the first and the second derivatives

(1 Mark)

$$f'(x) = 27x^2 - 4$$

$$f''(x) = 54x$$

f) Values of x where the function is increasing and decreasing

(1 Mark)

$$f'(x) = 0 \Rightarrow 27x^2 - 4 = 0$$

$$x = \pm \sqrt{\frac{4}{27}} = \pm \frac{2}{3\sqrt{3}}$$

$$\left(-\infty, -\frac{2}{3\sqrt{3}}\right), \quad \left(-\frac{2}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}\right), \quad \left(\frac{2}{3\sqrt{3}}, \infty\right)$$

$$f' > 0$$

$$f' < 0$$

$$f' > 0$$

Increasing on $\left(-\infty, -\frac{2}{3\sqrt{3}}\right) \cup \left(\frac{2}{3\sqrt{3}}, \infty\right)$

Decreasing on $\left(-\frac{2}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}\right)$

g) Values of x where the function is concave up and concave down

(1 Mark)

$$f'' = 0 \Rightarrow 54x = 0$$

$$\Rightarrow x = 0$$

$$\left(-\infty, 0\right) \mid \left(0, \infty\right)$$

$$f'' < 0 \mid f'' > 0$$

Concave up on $(0, \infty)$ and down on $(-\infty, 0)$

h) All minima and maxima and inflection points

(1 Mark)

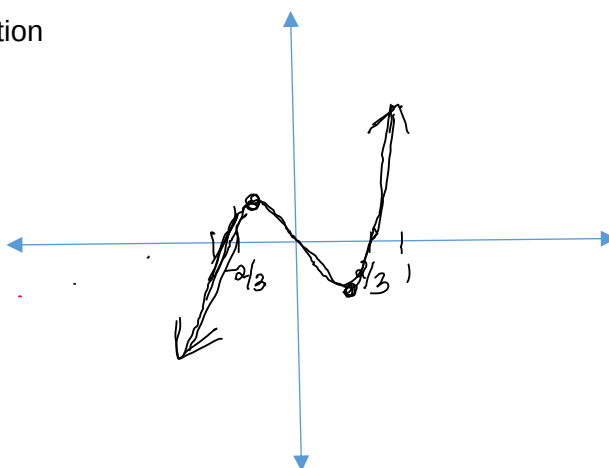
$$\text{Maximum} = f\left(\frac{-2}{3\sqrt{3}}\right) = \frac{16}{9\sqrt{3}}$$

$$\text{Minimum} = f\left(\frac{2}{3\sqrt{3}}\right) = -\frac{16}{9\sqrt{3}}$$

Inflection point (0,0)

i) Sketch the function

(1 Mark)



2. For the function $f(x) = \frac{-1}{\sqrt{9-x^2}}$, please find all of the following and then use that

information to sketch the curve. **(Steps may be out of order)**

a) All x and y intercepts.

(1 Mark)

b) Whether it is symmetric to the x-axis, y-axis or the origin

(1 Mark)

c) Asymptotes and behavior near the asymptotes

(1 Mark)

d) Domain and Range

(1 Mark)

e) Both the first and the second derivatives

(1 Mark)

f) Values of x where the function is increasing and/or decreasing

(1 Mark)

g) Values of x where the function is concave up and/or concave down

(1 Mark)

h) All minima and maxima and inflection points

(1 Mark)

Quiz 4

i) Sketch the function

(1 Mark)

3. A grocery store estimates that its daily profit P (in dollars) from producing x numbers of packaged Ready-To-Grab Food is $P = -0.05x^2 + 10x$. How many packages should be produced for maximum profit? (4 Marks)

4. A rectangular hole is to be cut in a wall for a vent. If the perimeter of the hole is 48 cm and the length of the diagonal is a minimum, what are the dimensions of the hole? (4 Marks)

5. Find the values of Δy and dy for the given values of x and dx . How close are the 2 values found? (Do not round off your answer) (4 Marks)

$$y = \frac{x}{\sqrt{6x-1}} \quad x = 3.50 \quad dx(\Delta x) = 0.025$$