

Quiz 4

Show all relevant steps for each question.

1. For the function $f(x) = 9x^3 - 4x$, please find all of the following and then use that information to sketch the curve. **(Steps may be out of order)**

- a) All x and y intercepts.

$$\underline{x\text{-int}} \quad f(x) = 0 \Rightarrow 9x^3 - 4x = 0$$

$$x(9x^2 - 4) = 0$$

$$x(3x+2)(3x-2) = 0$$

$$(0, 0), \left(-\frac{2}{3}, 0\right), \left(\frac{2}{3}, 0\right)$$

$$\underline{y\text{-int}} \quad f(0) = 0 \quad (0, 0)$$

- b) Whether it is symmetric to the x-axis, y-axis or the origin

$$f(-x) = -9x^3 + 4x = -f(x)$$

Symmetric to the origin.

- c) Behavior of the function as x becomes large

(1 Mark)

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

rises to the right and falls to the left.

- d) Domain and Range

(1 Mark)

$$D: (-\infty, \infty) \quad R: (-\infty, \infty)$$

e) Both the first and the second derivatives

(1 Mark)

$$f'(x) = 27x^2 - 4$$

$$f''(x) = 54x$$

f) Values of x where the function is increasing and decreasing

(1 Mark)

$$f'(x) = 0 \Rightarrow 27x^2 - 4 = 0$$

$$x = \pm \sqrt{\frac{4}{27}} = \pm \frac{2}{3\sqrt{3}}$$

$$\left(-\infty, -\frac{2}{3\sqrt{3}}\right), \quad \left(-\frac{2}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}\right), \quad \left(\frac{2}{3\sqrt{3}}, \infty\right)$$

$$f' > 0$$

$$f' < 0$$

$$f' > 0$$

Increasing on $\left(-\infty, -\frac{2}{3\sqrt{3}}\right) \cup \left(\frac{2}{3\sqrt{3}}, \infty\right)$

Decreasing on $\left(-\frac{2}{3\sqrt{3}}, \frac{2}{3\sqrt{3}}\right)$

g) Values of x where the function is concave up and concave down

(1 Mark)

$$f'' = 0 \Rightarrow 54x = 0$$

$$\Rightarrow x = 0$$

$$\left(-\infty, 0\right) \mid \left(0, \infty\right)$$

$$f'' < 0 \mid f'' > 0$$

Concave up on $(0, \infty)$ and down on $(-\infty, 0)$

h) All minima and maxima and inflection points

(1 Mark)

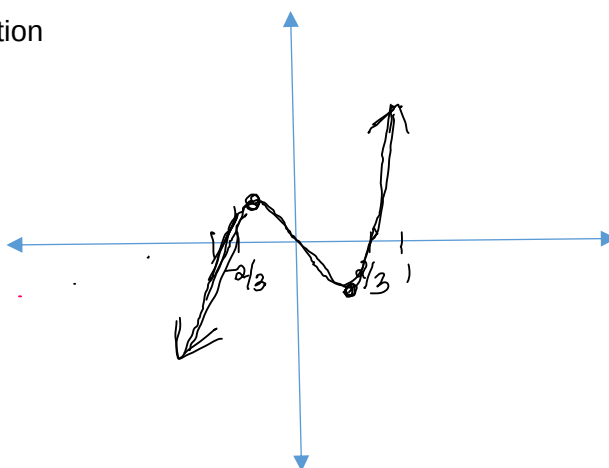
$$\text{Maximum} = f\left(\frac{-2}{3\sqrt{3}}\right) = \frac{16}{9\sqrt{3}}$$

$$\text{Minimum} = f\left(\frac{2}{3\sqrt{3}}\right) = -\frac{16}{9\sqrt{3}}$$

Inflection point $(0, 0)$

i) Sketch the function

(1 Mark)



2. For the function $f(x) = \frac{-1}{\sqrt{9-x^2}}$, please find all of the following and then use that information to sketch the curve. (Steps may be out of order)

a) All x and y intercepts.

(1 Mark)

X-int None

Y-int $f(0) = \frac{-1}{\sqrt{9}} = -\frac{1}{3} \quad (0, -\frac{1}{3})$

b) Whether it is symmetric to the x-axis, y-axis or the origin

(1 Mark)

$$f(-x) = \frac{-1}{\sqrt{9-x^2}} = f(x)$$

Symmetric to y.

c) Asymptotes and behavior near the asymptotes

(1 Mark)

Asymptotes $9 - x^2 = 0$
 $x = \pm 3$

$$\lim_{x \rightarrow -3^+} f(x) = -\infty \quad \lim_{x \rightarrow 3^-} f(x) = -\infty$$

d) Domain and Range

(1 Mark)

$$9 - x^2 > 0$$

$$D: (-3, 3) \quad R: (-\infty, 0)$$

e) Both the first and the second derivatives

(1 Mark)

$$f'(x) = \frac{-x}{(9 - x^2)^{3/2}}$$

$$f''(x) = \frac{(9 - x^2)^{3/2}(-1) - (-x) \cdot \frac{3}{2} \sqrt{9 - x^2} (-2x)}{(9 - x^2)^3}$$

$$= \frac{-2x^2 + 9}{(9 - x^2)^{5/2}}$$

f) Values of x where the function is increasing and/or decreasing

(1 Mark)

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$$f' = 0 \Rightarrow x = 0$$

$$(-3, 0) \quad (0, 3)$$

+

-

increasing on $(-3, 0)$
decreasing on $(0, 3)$

g) Values of x where the function is concave up and/or concave down (1 Mark)

$$f'' = 0 \Rightarrow -2x^2 - 9 \neq 0$$

$$f'' < 0 \text{ on } (-3, 3)$$

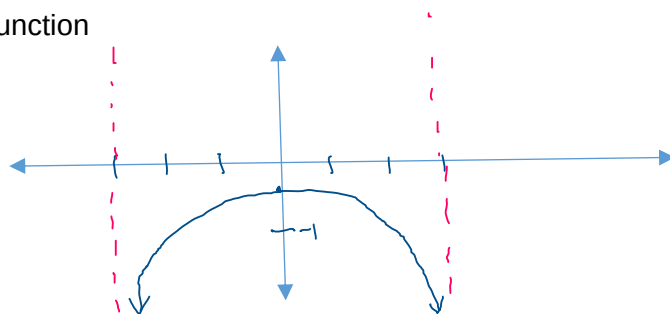
Concave down on $(-3, 3)$

h) All minima and maxima and inflection points (1 Mark)

Maximum $(0, -\frac{1}{3})$

No inflection points

i) Sketch the function (1 Mark)



3. A grocery store estimates that its daily profit P (in dollars) from producing x numbers of packaged Ready-To-Grab Food is $P = -0.05x^2 + 10x$. How many packages should be produced for maximum profit? (4 Marks)

$$P'(x) = -0.1x + 10 = 0$$

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$$x = \frac{10}{0.1} = 100$$

100 packages

4. A rectangular hole is to be cut in a wall for a vent. If the perimeter of the hole is 48 cm and the length of the diagonal is a minimum, what are the dimensions of the hole?

(4 Marks)

$$2x + 2y = 48$$

$$y = 24 - x$$

$$\begin{aligned} D = d^2 &= x^2 + y^2 \\ &= x^2 + (24 - x)^2 \end{aligned}$$

$$D' = 2x - 2(24 - x) = 0$$

$$4x = 48$$

$$x = 12$$

Dimensions: 12cm, 12cm

$$y = 12$$

5. Find the values of Δy and dy for the given values of x and dx . How close are the 2 values found? (Do not round off your answer)

(4 Marks)

$$y = \frac{x}{\sqrt{6x-1}}$$

$$x = 3.50$$

$$dx(\Delta x) = 0.025$$

$$\Delta x = f(3.5 + 0.025) - f(3.5)$$

$$= f(3.525) - f(3.5)$$

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$$\begin{aligned} &= \frac{3.525}{\sqrt{6(3.525-1)}} - \frac{3.5}{\sqrt{6(3.5)-1}} \\ &= 0.7853 - 0.7826 \\ &= 0.0027 \end{aligned}$$

$$dy = f'(x) dx$$

$$f'(x) = \frac{3x-1}{(6x-1)^{3/2}}$$

$$dy = f'(3.5)(0.025) = 0.0027$$

Very close error = 0.00001