

$$\begin{aligned}
 1) \quad & 5x - 3y + z = 7 \\
 & 2x + 3y - z = 0 \\
 & 8x + 9y - 3z = 2
 \end{aligned}$$

The augmented matrix is

$$\left[\begin{array}{ccc|c} 5 & -3 & 1 & 7 \\ 2 & 3 & -1 & 0 \\ 8 & 9 & -3 & 2 \end{array} \right] \begin{array}{l} \\ R_2 \rightarrow 5R_2 - 2R_1 \\ R_3 \rightarrow 5R_3 - 8R_1 \end{array}$$

$$\approx \left[\begin{array}{ccc|c} 5 & -3 & 1 & 7 \\ 0 & 21 & -7 & -14 \\ 0 & 69 & -23 & -46 \end{array} \right] \begin{array}{l} \\ R_2 \rightarrow R_2/7 \\ R_3 \rightarrow R_3/23 \end{array}$$

$$\approx \left[\begin{array}{ccc|c} 5 & -3 & 1 & 7 \\ 0 & 3 & -1 & -2 \\ 0 & 3 & -1 & -2 \end{array} \right] R_3 \rightarrow R_3 - R_2$$

$$\approx \left[\begin{array}{ccc|c} 5 & -3 & 1 & 7 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Here rank of the augmented matrix = rank of the coefficient matrix

$\therefore$ The system has infinite number of solution since the system is consistent

Now $5x - 3y + z = 7$ — (1)

$$3y - z = -2 \Rightarrow z = 3y + 2$$

From (1), $5x - 3y + 3y + 2 = 7 \Rightarrow x = \frac{7-2}{5} = 1$

$\therefore$ $x = 1$, y is free variable & $z = 3y + 2$