

2) Given that $A = \begin{bmatrix} 0.5 & -0.6 \\ 0.75 & 1.1 \end{bmatrix}$

Now, $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 0.5 - \lambda & -0.6 \\ 0.75 & 1.1 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (0.5 - \lambda)(1.1 - \lambda) + 0.6 \times 0.75 = 0$$

$$\Rightarrow 0.55 - 0.5\lambda - 1.1\lambda + \lambda^2 + 0.45 = 0$$

$$\Rightarrow \lambda^2 - 1.6\lambda + 1 = 0$$

$$\Rightarrow \lambda = \frac{1.6 \pm \sqrt{(1.6)^2 - 4 \cdot 1}}{2}$$

$$= \frac{1.6 \pm \sqrt{-1.44}}{2}$$

$$= \frac{1.6 \pm 1.2i}{2}$$

$$\therefore \lambda_1 = \frac{1.6 + 1.2i}{2} \text{ and } \lambda_2 = \frac{1.6 - 1.2i}{2}$$

$$\lambda_1 = 0.8 + 0.6i \text{ and } \lambda_2 = 0.8 - 0.6i$$

For $\lambda_1 = 0.8 + 0.6i$

$$(A - \lambda_1) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} 0.5 - (0.8 + 0.6i) & -0.6 \\ 0.75 & 1.1 - (0.8 + 0.6i) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0.3 - 0.6i & -0.6 \\ 0.75 & 0.3 - 0.6i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow (0.3 - 0.6i)x - 0.6y = 0 \quad \text{--- (1)}$$

$$0.75x + (0.3 - 0.6i)y = 0 \quad \text{--- (2)}$$

From (1)

$$(0.3 - 0.6i)x - 0.6y = 0$$

$$\Rightarrow y = \frac{(0.3 - 0.6i)x}{0.6}$$

let $x = \alpha$, α be any scalar

$$y = \frac{(0.3 - 0.6i)\alpha}{0.6}$$

$$\therefore (x, y)^T = \left(\alpha, \frac{(0.3 - 0.6i)\alpha}{0.6} \right)^T = \alpha \left(1, \frac{(0.3 - 0.6i)}{0.6} \right)^T$$

For $\lambda_2 = 0.8 - 0.6i$

$$[A - \lambda_2](x, y)^T = 0$$

$$\Rightarrow \begin{pmatrix} 0.5 - (0.8 - 0.6i) & -0.6 \\ 0.75 & 1.1 - (0.8 - 0.6i) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -0.3 + 0.6i & -0.6 \\ 0.75 & -0.3 + 0.6i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow (-0.3 + 0.6i)x - 0.6y = 0 \quad \text{--- (3)}$$

$$0.75x + (-0.3 + 0.6i)y = 0 \quad \text{--- (4)}$$

From (3),

$$y = \frac{(-0.3 + 0.6i)x}{0.6}$$

Let $x = \beta$, β be any scalar

$$y = \frac{(-0.3 + 0.6i) \cdot \beta}{0.6}$$

$$(x, y)^T = \left(\beta, \frac{(-0.3 + 0.6i)}{0.6} \beta \right) = \beta \left(1, \frac{(-0.3 + 0.6i)}{0.6} \right)$$

$$\text{Basis of } E_{\lambda_1} = \left\{ \left(1, \frac{0.3 - 0.6i}{0.6} \right) \right\}$$

$$\text{Basis of } E_{\lambda_2} = \left\{ \left(1, \frac{-0.3 + 0.6i}{0.6} \right) \right\}$$