MAT 2324, Differential Equations
Prof. Frithjof Lutscher, University of Ottawa
Winter 2021, Assignment #5
Due Sunday April 11, 9pm, online in brightspace, as a .pdf file of small size (less than 5MB, better yet, less than 2MB).

Note: The assignments in this course do not replace the practice problems in Bronson and Costa. The assignments cover material that is not covered in Bronson and Costa or not in that way. Assignments tend to be more "thinking" problems whereas the problems in Bronson and Costa tend to be more computational. I am also using the assignments to let you look just a little bit beyond the standard course material.

Note: Full marks require that your answer is mathematically and logically correct and well explained. You must convince me that you know why your solution is correct. Full marks also require that your assignment is written legibly and is reasonably well organized on the page.

Problem 1: Calculate e^{At} for

$$A = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -2 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

Problem 2: Find the solution to the following IVP by calculating the matrix exponential first and then using the variation of parameters formula:

$$\mathbf{x}' = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix} \mathbf{x} + e^{-3t} \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \qquad \mathbf{x}(0) = \mathbf{x}_0.$$

Problem 3: Consider the nonlinear planar system

$$x' = y(y-1)(y-2), \qquad y' = x(x+1)(x+2).$$

- Calculate all the nullclines and the direction of the vectorfield on the nullclines. Sketch your results in the phase plane. Please clearly indicate which the x and which the y nullcline is. If you choose colours, please use the same colour scheme as in the course notes.
- Calculate all steady states and the Jacobian matrix at those steady states.
- According to the linearization, what can you say about the stability and kind of the steady states? What can you not say?
- Use the website suggested in Chapter 18.5 of the course notes to generate a phase plane plot of the system and include enough solutions to make the dynamics visible. What can you tell from the plot that you cannot tell from the linearization?
- When you look at the image obtained by the online tool, what comes to mind? Can you find a fitting name for the image?

Problem 4: (Bonus) Consider the planar system

$$x' = x^2 - y^2, \qquad y' = x - p$$

with real parameter p. Do a complete analysis (nullclines, steady states, Jacobi matrix, stability, type of steady state). Your results will depend on parameter p. Illustrate your results by choosing two values of p for which the phase plane looks qualitatively different and sketch the nullclines, the vector field, and indicate the type and stability of the steady states.