

5 a)

b) $\rightarrow$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\det(A) = 1 \begin{vmatrix} 0 & -1 \end{vmatrix} - 2 \begin{vmatrix} -1 & -1 \end{vmatrix} + 3 \begin{vmatrix} 1 & 0 \end{vmatrix}$$

$$-1 + 4 + 3 = \underline{6}$$

hence A is a non zero matrix.

c) linear independence.

2 vector u and v are linearly independent if $xu + yv = 0$ $x=y=0$

lets take $u = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $v = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + w = \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix}$

A vector is said to be linearly independent if $xu + yv + zw = 0$

$$0 = x \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + w \begin{pmatrix} 3 \\ 4 \\ 4 \end{pmatrix}$$

$$x + 2y + 3w = 0$$

$$2x + 3y + 4w = 0$$

$$3x + 4y + 4w = 0$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(1)

$$-2 \begin{pmatrix} 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 & 0 \end{pmatrix} \quad \begin{array}{rrrr} -2 & -4 & -6 & 0 \\ + & 2 & 3 & 4 & 0 \\ \hline 0 & -1 & -2 & 0 \end{array}$$

$$\begin{array}{rrrr} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 3 & 4 & 4 & 0 \end{array}$$

$$-3 \begin{pmatrix} 1 & 2 & 3 & 0 \\ 3 & 4 & 4 & 0 \end{pmatrix} \quad \begin{array}{rrrr} -3 & -6 & -9 & 0 \\ + & 3 & 4 & 4 & 0 \\ \hline 0 & -2 & -5 & 0 \end{array}$$

$$\begin{array}{rrrr} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -2 & -5 & 0 \end{array}$$

$$\begin{array}{l} 2 \begin{pmatrix} 0 & -1 & -2 & 0 \end{pmatrix} \\ -1 \begin{pmatrix} 0 & -2 & -5 & 0 \end{pmatrix} \end{array}$$

$$\begin{array}{rrrr} 0 & -2 & -4 & 0 \\ + & 0 & 2 & 5 & 0 \\ \hline 0 & 0 & 1 & 0 \end{array}$$

$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$w = 0$$

$$-v - 2 = 0$$

$$-v - 0 = 0$$

$$-v = 0$$

$$v = 0$$

$$u + 2v + 3w = 0$$

$$u + 0 + 0 = 0$$

$$u = 0$$

hence the vectors are linearly independent

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 1 & 4 \end{pmatrix} \neq 0$$

(2)

$$1(12-4) - 2(8-12) + 3(2-9)$$

$$1(8) - 2(-4) + 3(-7)$$

$$8 + 8 - 21$$

$$16 - 21 = -5$$

$$\begin{vmatrix} 1 & 3 & 3 \\ 2 & 2 & 4 \\ 3 & 1 & 4 \end{vmatrix} = -5 \neq 0$$

d)

$$\text{Show } A^{-1} = \frac{\text{Adj}(A)}{\det(A)}$$

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$$

$$|A| = 1 - 2 = -1$$

$$-1 \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$$

$$\text{Adj}(A) = \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$$

$$\det(A) = -1$$

$$\frac{\text{Adj}(A)}{\det(A)} = \frac{\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}}{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$$

$$\text{Hence } A^{-1} = \frac{\text{Adj}(A)}{\det(A)}$$

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a) infinitely many solutions

b) Augmented matrix

$$\left(\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -2 & -3 \\ 1 & 1 & 1 & 2 & 2 \end{array} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ -1 & -1 & -1 & -2 & -3 & -2 \\ 1 & 1 & 1 & 2 & 2 & 3 \end{array} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

$$\begin{array}{r} -1 \\ 1 \end{array} \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ -1 & -1 & -1 & -2 & -3 & -2 \end{array} \right)$$

$$\begin{array}{r} + \\ 0 \end{array} \left(\begin{array}{ccccc|c} -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -2 & -3 & -3 \\ 0 & 0 & 0 & 1 & 2 & 2 \end{array} \right)$$

$$\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 2 & 2 \end{array}$$

$$\begin{array}{r} -1 \\ 1 \end{array} \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ -1 & -1 & -1 & -2 & -3 & -2 \end{array} \right) + \left(\begin{array}{ccccc|c} -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 2 & 2 & 2 \\ 0 & -2 & 0 & 1 & 1 & 1 \end{array} \right)$$

$$\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 2 & 2 \\ 0 & -2 & 0 & 1 & 1 & 1 & 1 \end{array}$$

$$\begin{array}{r} -2 \\ 0 \end{array} \left(\begin{array}{cccccc|c} 0 & 0 & 0 & 1 & 2 & 2 & 2 \\ 0 & 0 & -2 & 0 & 1 & 1 & 1 \end{array} \right)$$

$$\begin{array}{cccccc|c} 0 & 0 & 0 & -2 & -4 & -4 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & -4 & -4 & -4 \end{array}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & -2 & -4 & -4 \end{array} \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$$

4f

e)

- f) The solution set of a non-homogeneous matrix equation $A_{m \times n} \vec{x} = \vec{b}$ where $\vec{b} \neq \vec{0}$ is never a vector subspace of $\mathbb{R}^n$.
The solution set of homogeneous matrix equation $A_{m \times n} \vec{x} = \vec{0}$ is always a vector subspace of $\mathbb{R}^n$ very interestingly is called the null space.

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