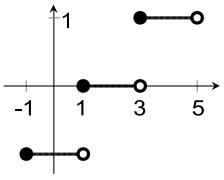
Problem 5 (8 points).

Suppose $g: \mathbf{R} \to \mathbf{R}$ where $g(x) = \left\lfloor \frac{x-1}{2} \right\rfloor$

a. Draw a graph of g

Solution:



b. Is g 1-1? (explain)

c. Is g onto $\mathbf{R}$? (explain)

d. Does $g^{-1}(x)$ exist? (explain)

Problem 6 (2 points).
Suppose $g: A \to B$ and $f: B \to C$, where $f \circ g$ is 1-1 and g is 1-1. Must f be 1-1?

Problem 7 (2 points).

Prove or disprove: For all positive real numbers x and y, $\lfloor x \cdot y \rfloor \leq \lfloor x \rfloor \cdot \lfloor y \rfloor$.

Problem 8 (2+2+2+2 = 8 points).

Suppose that $g : A \to B$ and $f : B \to C$, where $A = B = C = \{1, 2, 3, 4\}$, $g = \{(1, 2), (2, 3), (3, 2), (4, 4)\}$, and $f = \{(1, 2), (2, 4), (3, 2), (4, 3)\}$.

a. Find $f \circ g$

b. Find $g \circ f$

c. Find $g \circ g$

d. Find $g \circ (g \circ g)$

Problem 9 (2+2+2+2+2 = 10 points). Sequences
Give the sequence generated by the following formula for n = 1, 2, 3,…:

a. $a_n = 7$

b. $a_n = 2 - (-1)^n$

Find a formula that generates the following sequences for n = 1, 2, 3,…: (e.g. : $\mathbf{a_n = \ldots}$)

c. 5, 7, 9, 11, 13,…

d. 5, 5, 5, 5, 5,…

e. 7, 3, 7, 3, 7, 3,…

Problem 10 (2+2 = 4 points).

a. Find $\sum_{i=1}^{1000} i$

b. Find $\sum_{i=1}^{6}((-2)^i - 2^i)$