

CHAPTER 7

Configuration Design

LEARNING OBJECTIVES

When you have completed this chapter you will be able to

- Distinguish between product architecture and part configuration
- Apply guidelines for designing a product's architecture
- Synthesize alternative part configurations
- Utilize part configuration requirements sketches
- Design parts for greater ease of assembly
- Design parts using design for manufacture guidelines
- Evaluate alternative configurations

7.1 INTRODUCTION

As we finish the concept design phase, we determine one or more feasible concepts for the parts that go into our product. The concepts are merely abstract embodiments of physical principles, working geometries, and materials. We are eager to develop mathematical models, using our knowledge of the sciences and mathematics. We might also want to build a scale model for testing. But we can't do that just yet, because we have no specific details such as critical sizes or specific materials. Most likely, we have not even considered which of the components should be standard and therefore purchased.

For example, let's consider the design of a transmission, which transforms rotational speed and torque. Assume that we selected a gear pair, shown in Figure 7.1, as the leading concept for our design, rather than a chain drive or belt-and-pulley system. We would like to proceed with parametric design and analyze gear tooth bending stresses, shaft deflections, bearing loads, and so on. But to set up the equations, or perform the computer analyses, or build a scale model we need some more information. In particular, we need to examine configuration alternatives.

The gear-pair concept design is deliberately fuzzy. As we examine the figure, we realize that much of what remains unknown deals with configuration, such as:

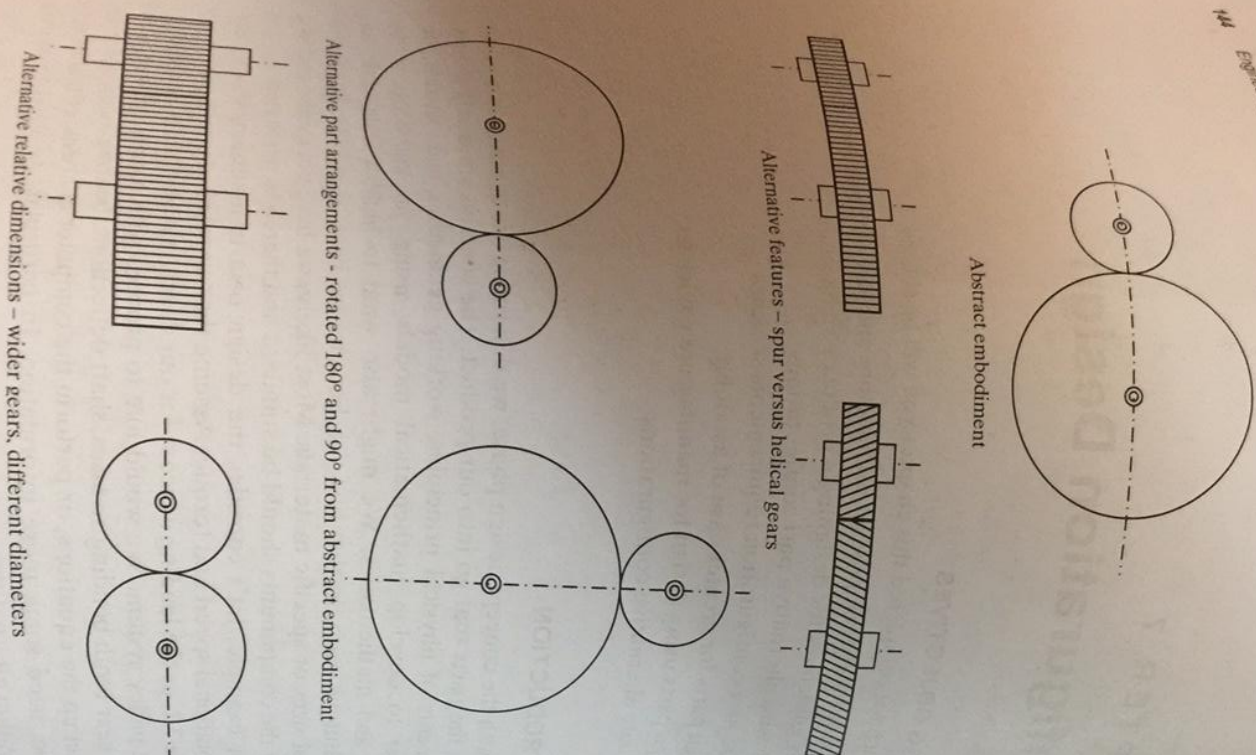


FIGURE 7.1 Alternative configurations for a gear-pair transmission.

- What type of teeth should the gears have, spur or helical?
- Should the pinion be located to the left of the gear, right, above, or below?
- What size should the diameters be relative to each other?
- What type, size, and location should we consider for the shaft bearings?
- How will we attach the drive pinion and output gear to the shaft?

Configuration design deals with **product architecture**, which is the selection and arrangement of components on a product, and **part configuration**, which is the selection and arrangement of features on a part.

When **configuring a product**, our primary concern is architectural, that is, determining the number and type of components (standard, special, part, or subassembly), their specific function, how they are spatially arranged, and how they are interconnected. When configuring the gear-pair, we could choose different types of standard gears (helical, bevel, or spur), rearrangements of axes (horizontal, vertical, parallel, perpendicular), and different thicknesses and diameters. We could choose standard roller, tapered roller, or ball bearings to support the shaft. We could custom-design the shaft or specify a standard shaft with machined keyway. Product configuration design helps us decide what we buy versus what we design and manufacture as special-purpose parts. By considering product architecture we can also examine integrating separate components into multifunction components.

When **configuring a special-purpose part**, our goal is to determine the type and number of geometric features, their arrangement or **connectivity**, and the relative dimensions of selected features. For example, let's examine the custom-designed bracket shown in Figure 7.2. We can create alternative configurations by adding or removing features such as walls, holes, fillets, slots, and chamfers. In addition, we can change their arrangements and relative dimensions to generate additional configurations. We should also create a list of unknown design variables such as hole diameter (D), wall thickness (t_w), fillet radius (R), width (W), and chamfer thickness (t_c). We will determine values for these variables later, in parametric design.

When **"configuring" a standard component** (i.e., part or subassembly), our goal is to select the type of component and its relative dimensions. For example, in selecting a rolling element bearing, which is a standard subassembly, we would select one of the following types: ball bearings, roller bearings, needle bearings, and tapered roller bearings. Additionally, we would consider the bearing width relative to its height, and prepare a list of unknown design variables, such as bore diameter and bearing width.

Configuration design, therefore, is the phase of product development when we determine the number and type of parts or geometric features in our design, how they are spatially arranged or interconnected, and approximate relative dimensions of the parts or features, and develop a list of design variables, as summarized in Table 7.1.

By generating a broad selection of alternative configurations, we systematically investigate different possibilities. Then, we can be selective, and

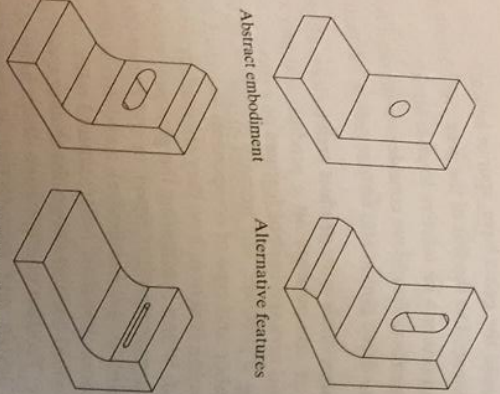


FIGURE 7.2 Alternative bracket configurations.

TABLE 7.1 Configuration Decisions for Different Product Elements

Configuration Problem	Required Decisions
Product	Type of component(s) Number of components Arrangement / connectivity
Special purpose part	Type of geometric features Number of geometric features Arrangement / connectivity
Standard component (part or subassembly)	Relative dimensions Name(s) of unknown design variable(s) Specific type Names of unknown design variable(s)

choose the best configuration to develop further. In other words, to be selective we need a selection. We will spend time and energy preparing different configurations. Many may not be used. But, by proceeding in a systematic way, we are likely to develop one or more configurations that meet our requirements.

To be selective, we need a selection.

In the remaining sections we will examine methods to systematically generate, analyze, and evaluate alternative configurations as shown in Figure 7.3.

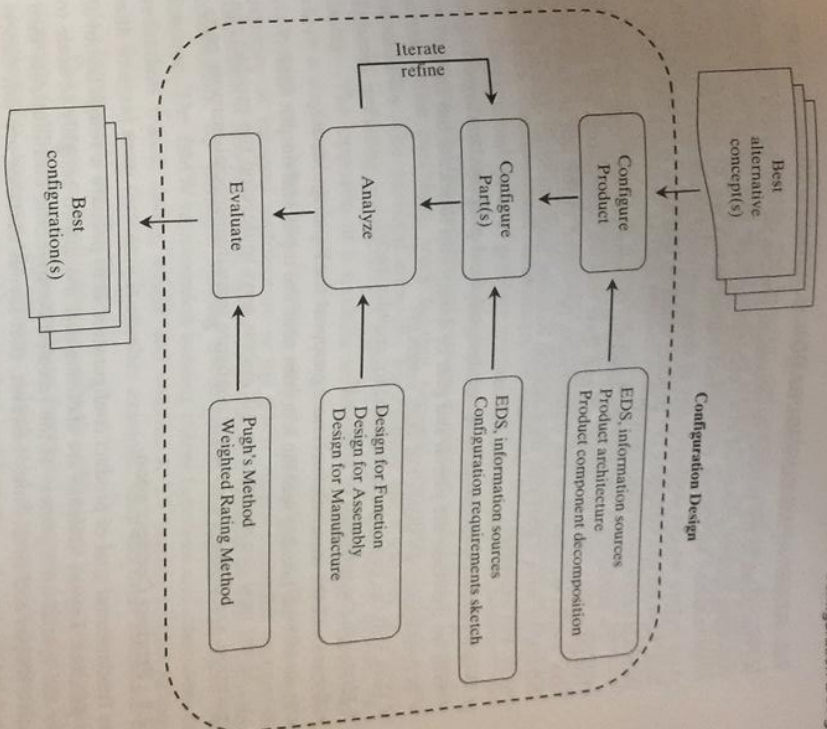


FIGURE 7.3 Configuration design from product to part.

First, we will consider how the product architecture can be configured. Then we will examine how parts can be configured, then analyzed and refined using DFA and DFM guidelines. The refined configurations are then evaluated to determine their relative merit so that we can select one or two for parametric design.

Product development is not a simple and/or linear decision-making process. Product development is characteristically complex. It often deals with difficult problems and complicated by the multitude of stakeholders having their say. The simplified models and diagrams presented thus far, however, give us a general framework to better understand and deploy the collection of better design methods used in industry. We recognize that companies will also use their own methods and procedures that have been developed and fine-tuned to meet their particular needs.

7.2 GENERATING CONFIGURATION ALTERNATIVES

At this point in the product development process we will have a collection of concepts, loosely conceived as a "product concept" that fulfills our customer's requirements. Of course, the abstract embodiments are, by nature, undefined requirements. And before we can configure any of these, we need to configure the product. But, how do you configure the product when all you have is a collection of ill-defined concepts or components?

To configure a product we consider the number and type of concepts (components), their arrangement, and their relative dimensions. For example, how would we configure the transmission presented in the last section? Even though we have a gear-pair transmission concept, many product details remain fuzzy. For example:

- Will we select standard bearings or design special-purpose bearings?
- Will we purchase the pinion and gear or design and fabricate our own?
- Where should we locate the shaft bearings?
- How will the bearings be removed for maintenance?
- How will the lubricant be replaced?
- Where will the transmission housing support the bearings loads?
- How will the housing seal the lubricant near the input and output shafts?

In the next two sections we will discuss ways to develop the product configuration, and then, how we might configure individual special-purpose parts.

7.2.1 Product Configuration

The fundamental goal of product configuration is to prepare a rough layout of the product. **Layout sketches** are sketches that are drawn roughly to scale to show the spatial arrangement of the selected components and illustrate their relative shapes and sizes. Note, however, that layout sketches are not dimensioned or toleranced.

Before we draft our first layout sketch we should review the engineering design specification for the product, along with any other pertinent documentation. Of particular importance we need to consider the:

- spatial limitations with respect to overall width, height, and depth
- product interactions with other physical objects or the user
- repair and cleaning, adjusting, and lubricating of specific components
- wear of components
- company's or customer's desire to customize the product configuration
- need to include standard parts or assemblies
- need to design parts to any specific dimensional or industry standards
- need to replace consumable materials.

Next, we should review sources of good ideas, including: company archives, co-workers, experts, the Internet, and creative methods. We might consider a similar product layout to get us started with our own. In addition, we could acquire and disassemble some example products to help define alternative product architectures.

At this point, we can develop the architecture of the product. As proposed by Ulrich and Eppinger (1995), product architecture is how physical elements and functional elements of a product are arranged or **clustered** into physical building blocks called **chunks** and how the chunks interact. The **physical elements** are the physical concepts developed during the conceptual design phase, and/or the components such as standard or special-purpose parts and/or assemblies. The **functional elements** are the functions that a product performs.

Two general types of product architecture are: modular architecture and integral architecture. In a **modular architecture**, chunks implement one or a few functions, and the interactions between chunks are well defined. Examples include office partitions, track lighting, personal computers, flashlights, and automobiles. The personal computer for example has a number of physical elements including: motherboard, power supply, case, keyboard, mouse, and display. The opposite of a modular architecture is an **integral architecture**, where a single chunk implements many functional elements or many chunks implement or share one function, and the interaction is ill-defined. An example of integral architecture is the BMW model R1100RS motorcycle transmission housing which serves as a structural element as well as driveline component. By **function sharing** a component can fulfill more than one function. The BMW transmission housing shares functions and eliminates the redundant frame, saving weight and volume. A TV remote control housing with integral snap fasteners is another example. Snap fasteners eliminate the need for separate metal screws.

The process of configuring a product's architecture is to cluster physical elements (components) and remaining functional elements into chunks, then geometrically arrange the chunks. Ulrich and Eppinger recommend the following procedure to develop a product architecture:

1. *Create a schematic* of the products elements, including physical elements that the team has agreed upon and those functional elements that have not yet been reduced to a physical concept. Keep the number of elements under 30. Separate the others to be worked on later.
2. *Cluster elements into chunks*. Start with a single element and cluster it with another when it is beneficial to:
 - share functions
 - enable standardization (use or create standard parts or modules)
 - integrate geometry (i.e., dimensional precision between features)
 - exploit vendor capabilities (e.g., such as injection molding expertise)
 - fully utilize manufacturing process capabilities
 - accommodate consumer variety (options, customization)

- exploit common interfaces (120 VAC, air duct connectors, PC bus)
 - manage changing technologies or components
 - provide for wear, upgrades, maintenance, consumables
3. *Make a rough geometric layout.* Use either 2D or 3D sketches (isometric or axonometric pictorials) and/or CAD drawings or build a physical model, type. Arrange chunks so that the interfaces remain feasible. Recluster any element to maintain feasible interactions and interfaces. Discuss the layout with others on the team. Refine the layout as necessary.
 4. *Identify the fundamental and incidental interactions.* Examine the flow of matter, energy, and signal between chunks. Are the desired functions preserved? Identify incidental interactions such as unwanted vibrations, or heat generation, or EMI, and show on incidental interaction graph.
- An example of a desktop printer architecture is shown in Figures 7.4a–d.

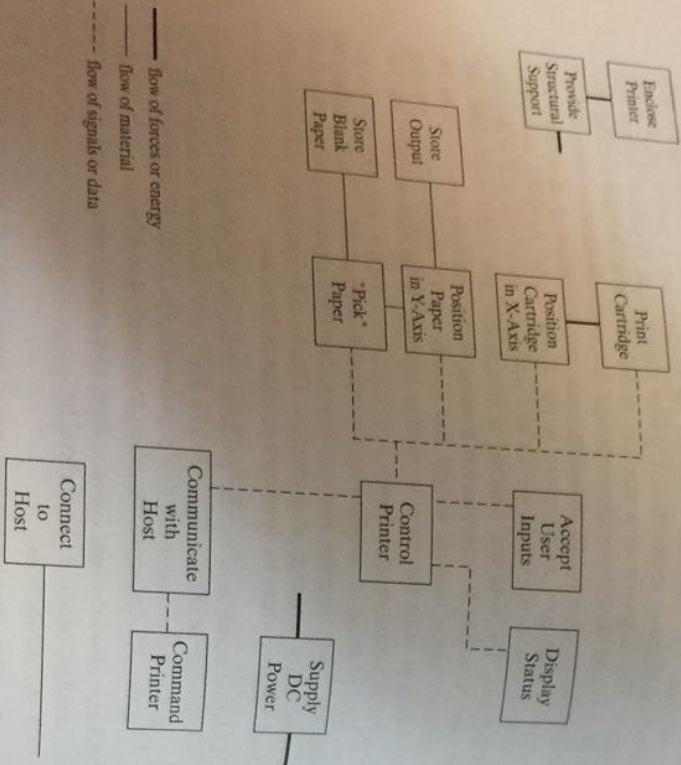


FIGURE 7.4a Product architecture schematic of elements. (K. T. Ulrich and S. D. Eppinger, *Product Design and Development*, Copyright 1995, McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.)

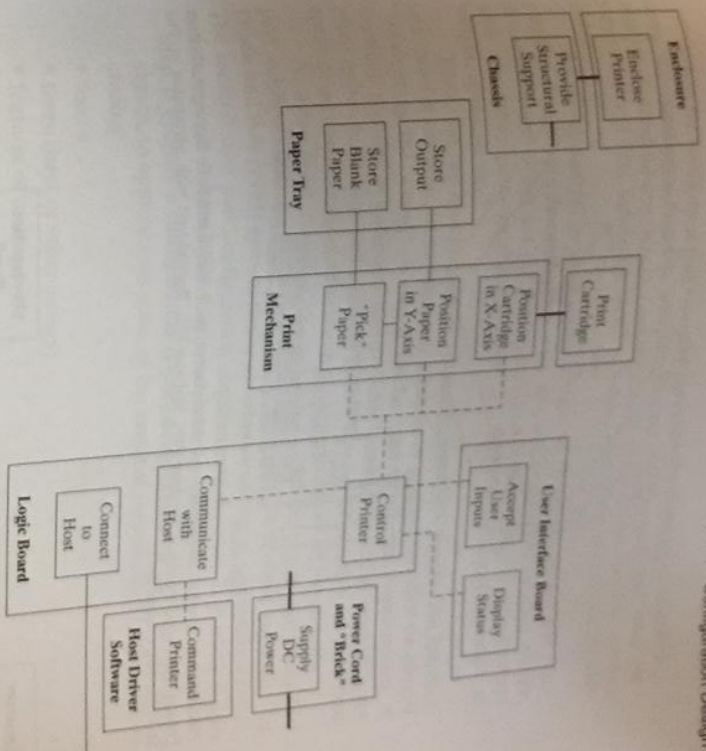


FIGURE 7.4B Product architecture—clustered diagram. (K. I. Ulrich and S. D. Eppinger, Product Design and Development, Copyright 1995, McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.)

7.2.2.2 Part Configuration

A **part** is a single-piece component that is made out of one material and needs no assembly. A part has geometric **features** such as: walls, ribs, projections, fillets, bosses, rounds, cylinders, tubes, cubes, cones, spheres, holes, slots, notches, chamfers, and grooves. The **arrangement** of features includes both locations and orientations. When features are connected to each other we say they are contiguous features. Features, also, have different widths and heights with respect to each other, which we call **relative dimensions**. A "tall" feature would be high relative to its width. A needle bearing would have a very small diameter relative to its length, whereas a roller bearing has comparable diameter and length.

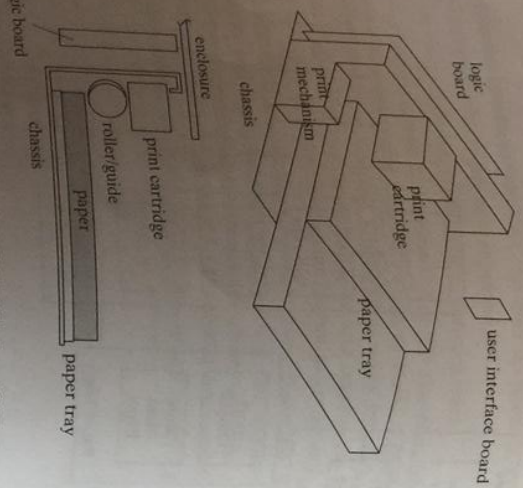


FIGURE 7.4c Product architecture—geometric layout. (K. T. Ulrich and S. D. Eppinger, *Product Design and Development*, Copyright 1995, McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.)

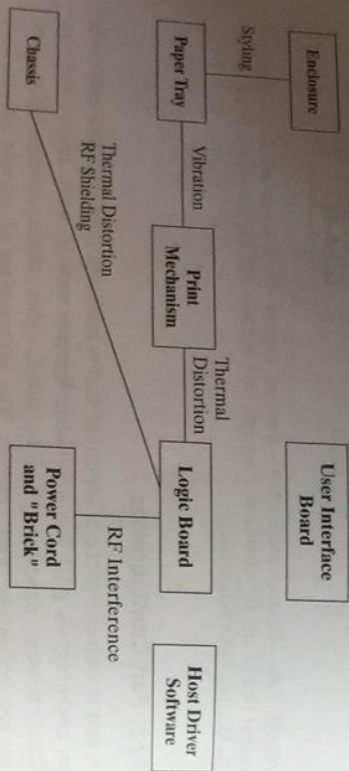


FIGURE 7.4d Product architecture—incidental interaction diagram. (K. T. Ulrich and S. D. Eppinger, *Product Design and Development*, Copyright 1995, McGraw-Hill. Reproduced with permission of The McGraw-Hill Companies.)

When we configure a part we make decisions about the types of part features, the number of features, how the features are arranged, and what the relative dimensions are. We also establish a list of the design variable names. Later, we will find values for those variables during the parametric design phase.

As mentioned for product configuration, we should reexamine the engineering design specification along with any other pertinent product design documentation. What are the principal loads and how does the part interact with the other parts in the assembly? Are there any constraints, such as weight, space, or size?

Then, too, we should consider researching archives, people, the Internet, and creative methods, to stimulate our thinking, or to make sure that we are not "reinventing the wheel." Would a similar part satisfy the requirements, if modified slightly? Is a standard part available? Standard parts, in particular, are often quite economical and available from a number of suppliers. A little time spent doing some background research can sometimes pay off handsomely.

At some point, however, the pencil has to hit the paper and alternative part configuration sketches need to be prepared. A method proposed by Dixon and Poli (1995) systematically generates a variety of part configurations. The method uses part configuration requirements sketches. It is fairly simple and focuses on essential-part requirements.

Step 1: Prepare a **configuration requirements sketch** for the part, which is a sketch, to approximate scale, of the essential surroundings of the part, including:

- forces
- flows (heat or other energy)
- features of mating parts
- support points or areas
- adjacent parts obstructions or forbidden areas

Note that the requirements sketch does not show any features of the part to be configured, only its surroundings. We make a number of photocopies of the configuration requirements sketch. Let's assume we make two copies.

Step 2: Prepare a few alternative **noncontiguous part configurations**. We add a partial wall or boss to the configuration requirements sketch at *mating or coupling points or surfaces*. The partial wall or boss is not connected to anything else, except the mating point or surface. It is called a "noncontiguous" feature. Set that noncontiguous configuration aside for the moment. Take a fresh configuration requirements sketch and add a partial wall or boss to a different point or area that is feasible. Set this alternative noncontiguous configuration sketch aside. And so on. Make a couple photocopies of each of the noncontiguous configurations. Let's assume we drew two noncontiguous sketches and that we make three copies of each.

Step 3: Prepare alternative **contiguous configurations**. We take one of the alternative noncontiguous sketches and connect the noncontiguous walls or bosses with material, as a single contiguous part. We do the same with the remaining five alternatives.

Step 4: Refine the configuration sketches. We improve the sketches by considering how the part will perform, how it will be manufactured, the material usage, and how it will be assembled. In this example we refine the six sketches. As we can see, even in this simple example, a variety of alternatives can be systematically generated with this method.

Example

Let's consider the configuration design of a kitchen sink sponge holder. The user inserts a soapy sponge into the holder, letting it drain and air-dry. The holder can also be used to drain and dry pot-scrubbing pads. The concept design includes a box-like container that fits into a suction cup that is attached to the kitchen sink. A thermoplastic material and injection molding is anticipated for the first run of about 50,000 units. The force from the suction cup is expected to support the box and sponge. The wet sponge exerts a downward force. Prepare two or more contiguous part configuration sketches.

As shown in Figure 7.5(a), the basic concept is a box supported on the side of a sink shown as a cutaway in Figure 7.5(b). We prepare a configuration requirements sketch, Figure 7.5(c), showing the downward sponge force and the upward suction cup force, along with a rough scale dotted line for the sink wall. We then add two chunks of material where the forces are applied. Figure 7.5(d) becomes our noncontiguous part configuration sketch.

We then prepare contiguous configurations as shown in Figure 7.6 (a-d). The container side walls have not been shown to simplify the force flow illustration. Configuration (a) shows a basic connection from the suction cup pin to the bottom of the container. This exposes much of the suction cup, which is better hidden in configuration (b).

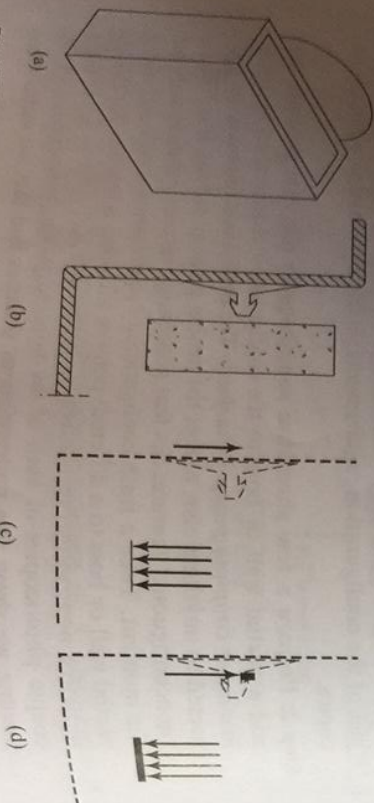


FIGURE 7.5 Development of a non-contiguous part configuration sketch for a sponge holder.

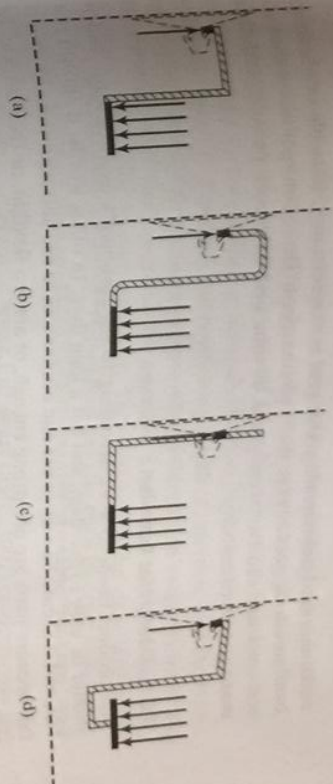


FIGURE 7.6 Alternative contiguous configuration sketches for a sponge holder.

Then in configuration (c) we see that the back wall can be pierced to eliminate material. Alternative (d) provides a drain trough. These four feasible configurations can now be analyzed, evaluated, and refined, as discussed later in this chapter.

7.3 ANALYZING AND REFINING CONFIGURATION ALTERNATIVES

When analyzing configuration alternatives we consider the likelihood that the parts will function as expected, that the parts can be assembled, and that they can be manufactured. We use our knowledge about working principles, materials, and manufacturing processes to logically reason whether the configurations will be satisfactory. We typically do not, however, prepare engineering analyses, such as comprehensive calculations, for any of the alternative configurations we have generated. We may sometimes, however, prepare some back-of-the-envelope, first-order estimates. The reason we wait is because comprehensive performance calculations are completed during parametric design. And, since that will require a lot of work, we would, first, like to reduce the number of alternative configurations, by screening out some of the less prospective candidates. Then, we will evaluate the remaining successful configurations using Pugh's method or the weighted-rating method to select the best one or two for the parametric design phase.

To "logically reason the likelihood," we develop a configuration "checklist" using three criterion categories:

- Design for function (will the configuration likely work?), and
- Design for assembly (will it likely assemble?), and
- Design for manufacture (can it be likely fabricated?)

The checklist is used to systematically review each configuration to determine whether the candidate is likely to perform the required function, assemble and be fabricated. We use our experience and knowledge of the basic sciences to

make this gut-level screening. Our goal is to screen out infeasible or eliminate configurations, not to judge their relative merit. Those configurations that are screened out can be reconfigured, if time and resources permit, and then reanalyzed for feasibility.

7.3.1 Design for Function

The product needs to work according to the customer's expectations, and it should be easy to maintain, and last a long time. In other words, it should not fail, and it should be easy to use and economical to use and maintain. Products fail because parts are not strong enough, or are too flexible, or they buckle, or jam due to thermal expansion. Their performance can degrade over time because of wear and/or corrosion. They also become uneconomical because they use too much fuel or energy or cost too much to repair and be disposed. These items are listed, along with a few others relating to function, in Table 7.2.

Table 7.2 Design for Function checklist items

1. Strong	11. Reliable
2. Stiff or flexible	12. Human factors/ergonomics
3. Buckle resistant	13. Accessible
4. Thermal expansion	14. Safe
5. Vibrate	15. Easy to use
6. Quiet / Noise	16. Maintainable
7. Heat transfer	17. Repairable
8. Fluids transport / storage	18. Durable (wear, corrosion)
9. Energy efficient	19. Life-cycle costs
10. Stable	20. Styling/aesthetics

7.3.2 Design for Assembly

Design for assembly (DFA) is the name used to describe a set of practices that aim to reduce the costs of part handling, insertion, and fastening. Product components should be easy to handle, insert, or mate, and fasten together. This is true whether the product is assembled manually or automatically. When **handling** a part we typically grasp it from a storage bin or location, move it to a new location for mating with the rest of the assembly, and correctly orient the part. Then we insert or mate the part and fasten it. Boothroyd et al. (1994) have shown that part handling is largely influenced by part features or attributes including: part symmetry, size, thickness,

weight, nesting, tangling, fragility, flexibility, temperature, slipperiness, stickiness, and the need for two hands, tools, or optical magnification. And further, these researchers have shown that insertion and fastening are influenced by factors including: accessibility, resistance (force) to insertion, visibility, ease of alignment and positioning, depth of insertion, and the type of fastener used. The researchers have also developed an excellent method to estimate the time required to assemble a product and evaluate the product's assembly design efficiency. The method can be done manually using tables and spreadsheets. The method has also been incorporated into a computer software package that automates many of the table "lookups" and calculations.

The Society of Manufacturing Engineers (SME, 1992) recommends the following design for assembly principles or guidelines, including:

- ✓ minimizing part count
- ✓ encouraging modular assembly
- ✓ designing parts with self-fastening features (snap-fits, press-fits)
- ✓ using standard parts
- ✓ stacking subassemblies from the bottom up
- ✓ designing parts with self-locating features (e.g., chamfers, aligning recesses/dimples)
- ✓ eliminating reorientation (i.e., insertion from two or more directions)
- ✓ facilitating parts handling (grasp, orient, move)
- ✓ minimizing levels of assembly (number of assemblies)
- ✓ eliminating (electric) cables

Alternatively, we can use graphical guidelines as shown in Figures 7.7a-d (Otto and Wood, 2001).

As we reflect on the above design for assembly recommendations, we find that many of them deal with varying the product architecture and part interface configurations. For example, reviewing the SME guidelines we vary product architecture by (1) encouraging modular assembly, (2) using standard parts, (3) stacking from bottom up, and (4) minimizing the number of parts. Similarly, we vary part interface configurations by (1) self-fastening features, (2) self-locating features, and (3) eliminating reorientation.

In summary, we develop a checklist of important DFA criteria from the handling and insertion factors reported by Boothroyd et al., the principles proposed by the Society of Manufacturing Engineers, and the Otto & Wood illustrations. Also note that we will often make small revisions to our candidate configurations during the screening process.

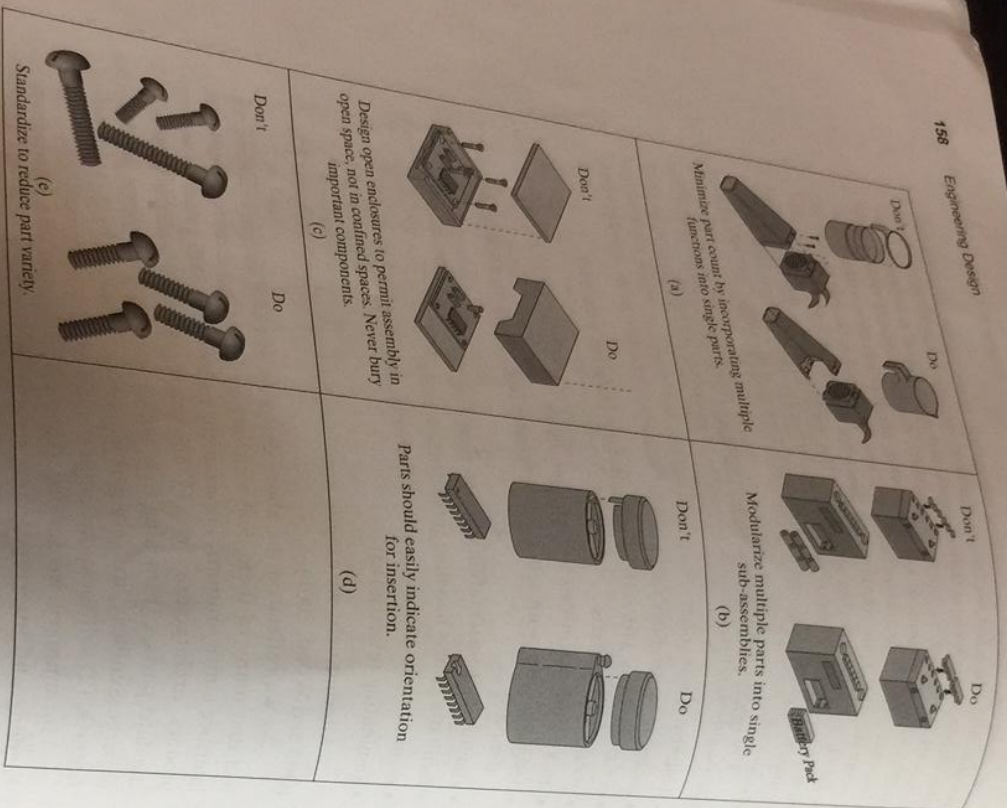


FIGURE 7.7a Design for assembly examples. (Product Design: Techniques in Reverse Engineering and New Product Development, by Otto/Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

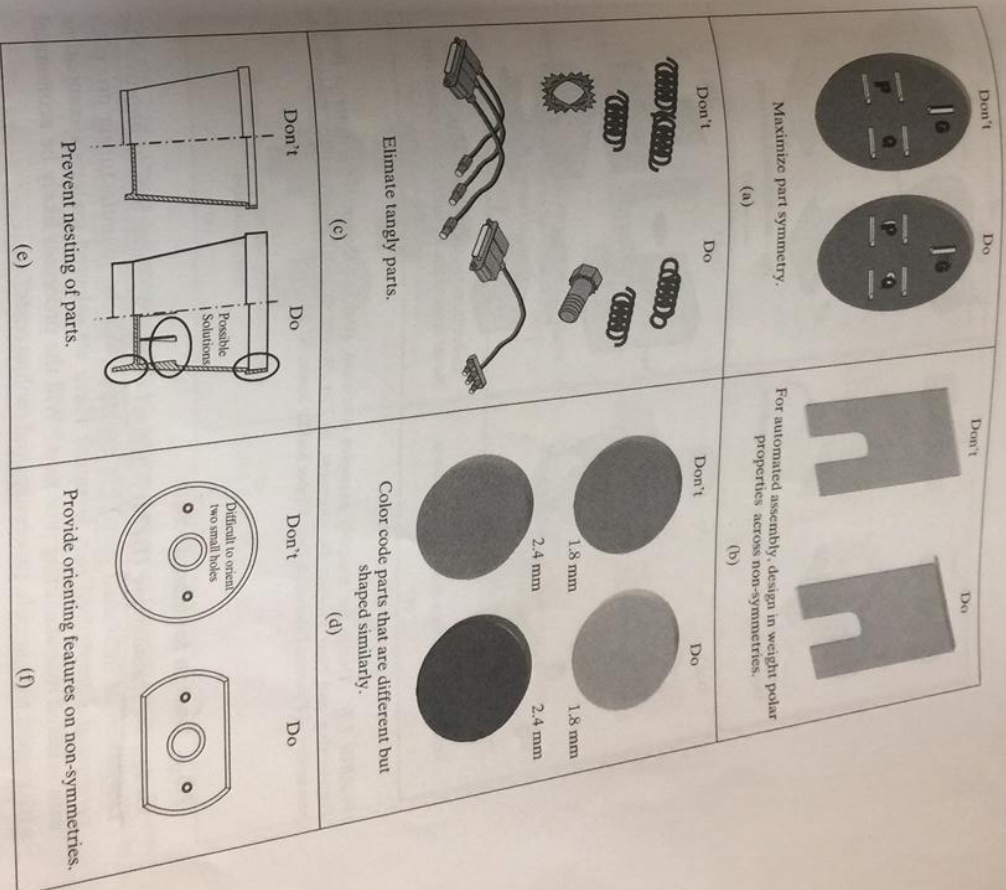


FIGURE 7.7b Design for assembly examples continued. (Product Design: Techniques in Reverse Engineering and New Product Development, by Otto/Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

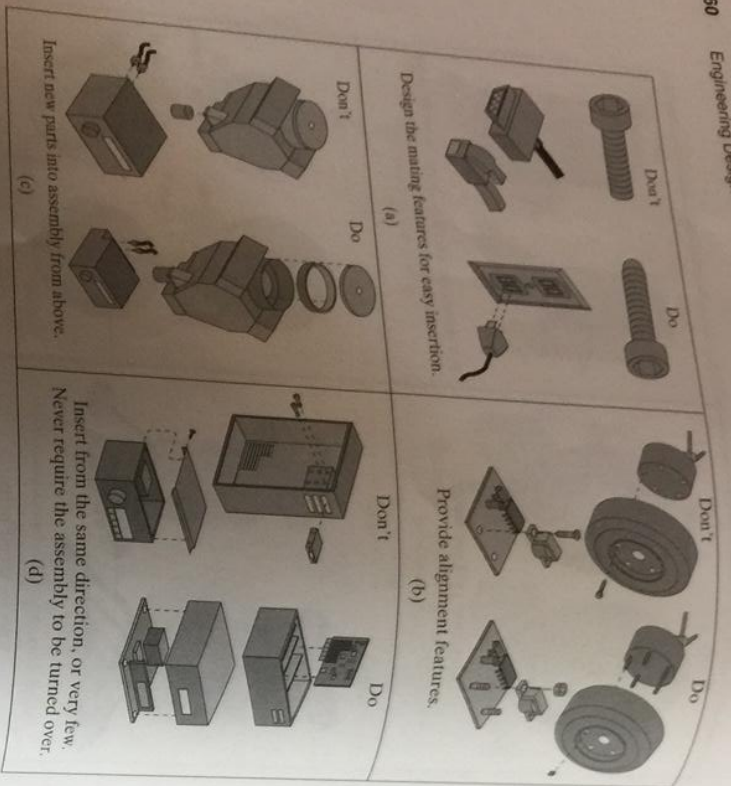


FIGURE 7.7c Design for assembly examples continued. (Product Design: Techniques in Reverse Engineering and New Product Development, by Otto Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

7.3.3 Design for Manufacture

Design for manufacture (DFM) is the set of practices that aim to improve the fabrication of individual parts. For example, will the manufacturing processes produce the configured part features? Have we selected materials compatible with selected manufacturing processes? Will the processes likely be economical with respect to materials, processing, and tooling costs?

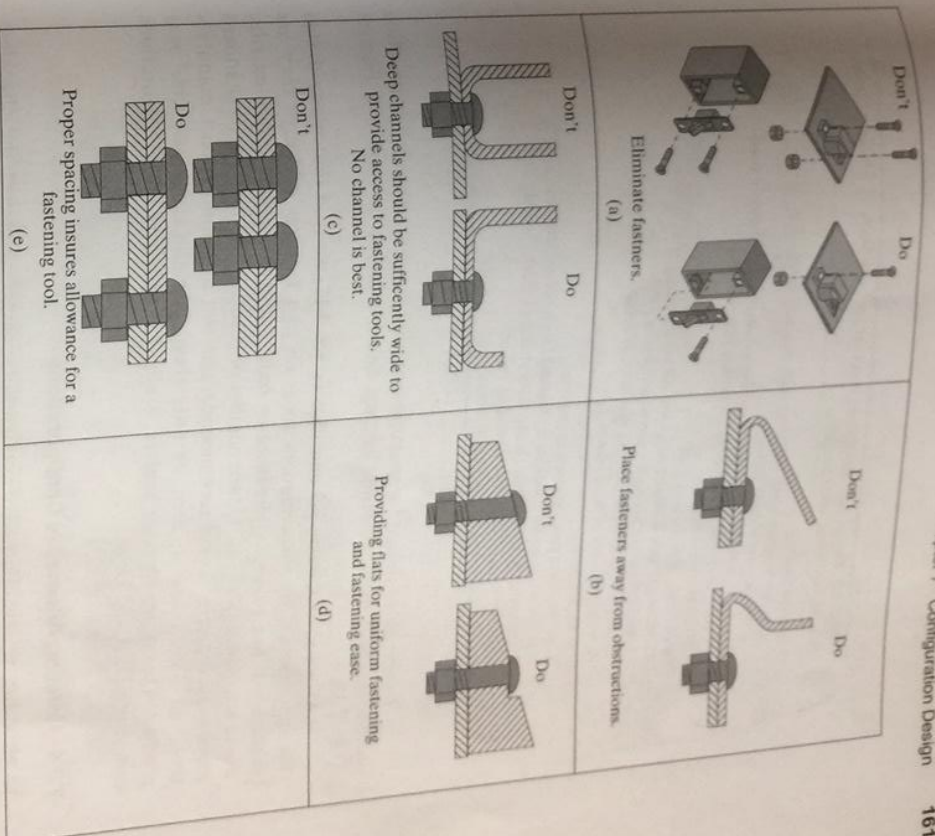


FIGURE 7.7d Design for assembly examples continued. (Product Design: Techniques in Reverse Engineering and New Product Development, by Otto Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

To screen alternative-part configurations for their manufacturability, we can develop a set of design for manufacture guidelines or checklist. Table 7.3 is a checklist for parts made by molding/casting, sheet metalworking, and machining. It was prepared by reviewing the capabilities of the manufacturing processes presented earlier in the text.

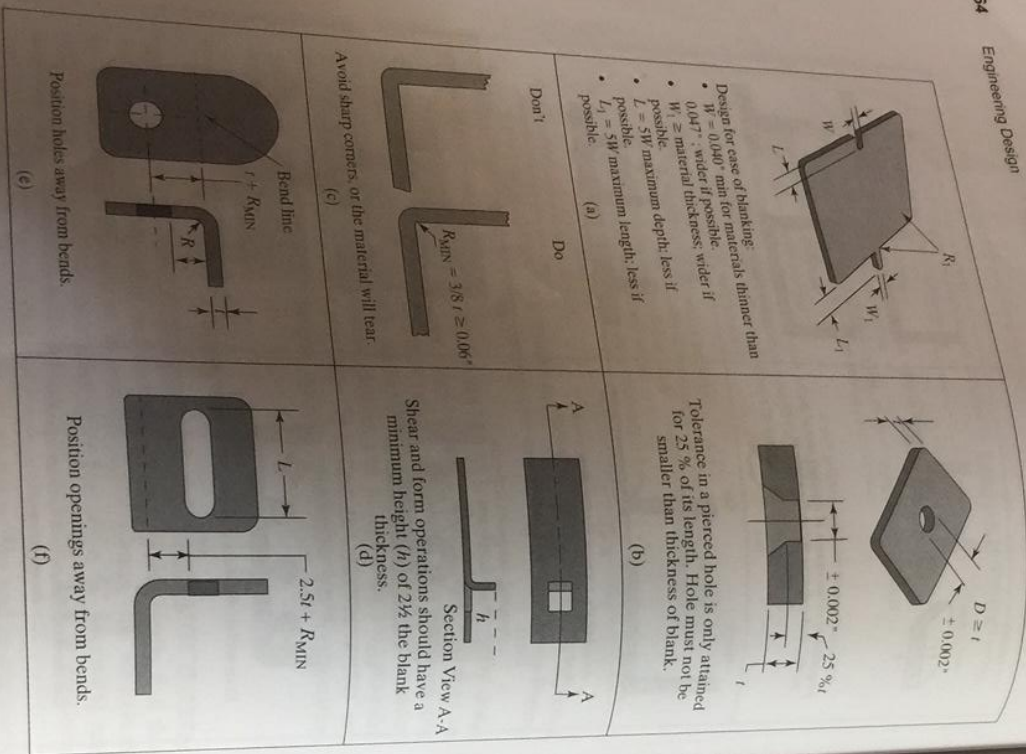


FIGURE 7.9 Design for manufacture of sheet metal parts. (Product Design: Techniques in Reverse Engineering and New Product Development, by Otto Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

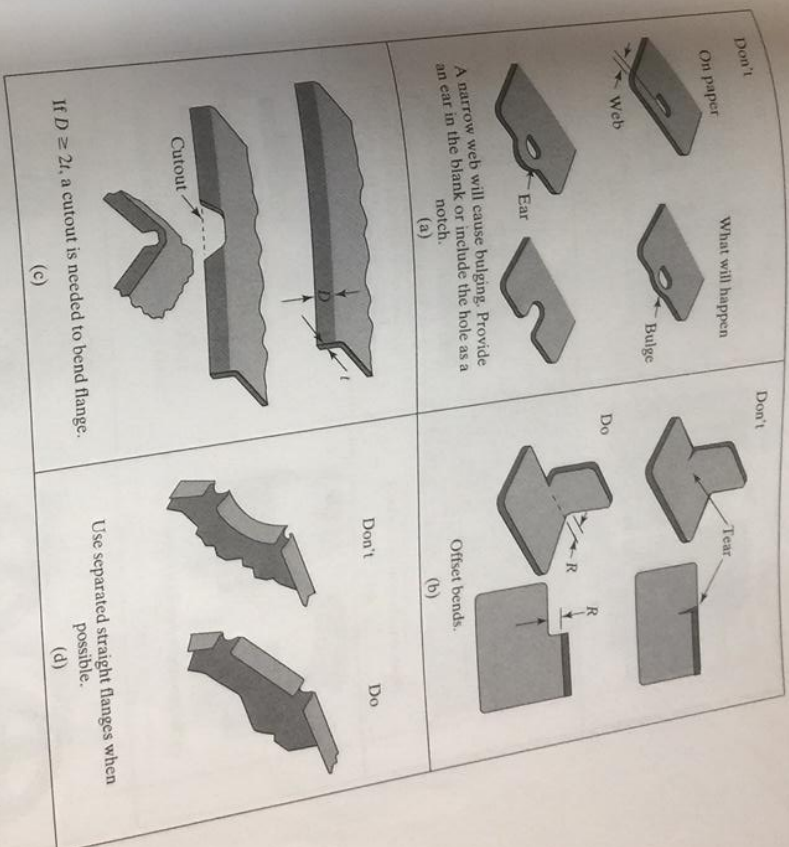


FIGURE 7.10 Design for manufacture of sheet metal parts continued. (Product Design: Techniques in Reverse Engineering and New Product Development, by Otto Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

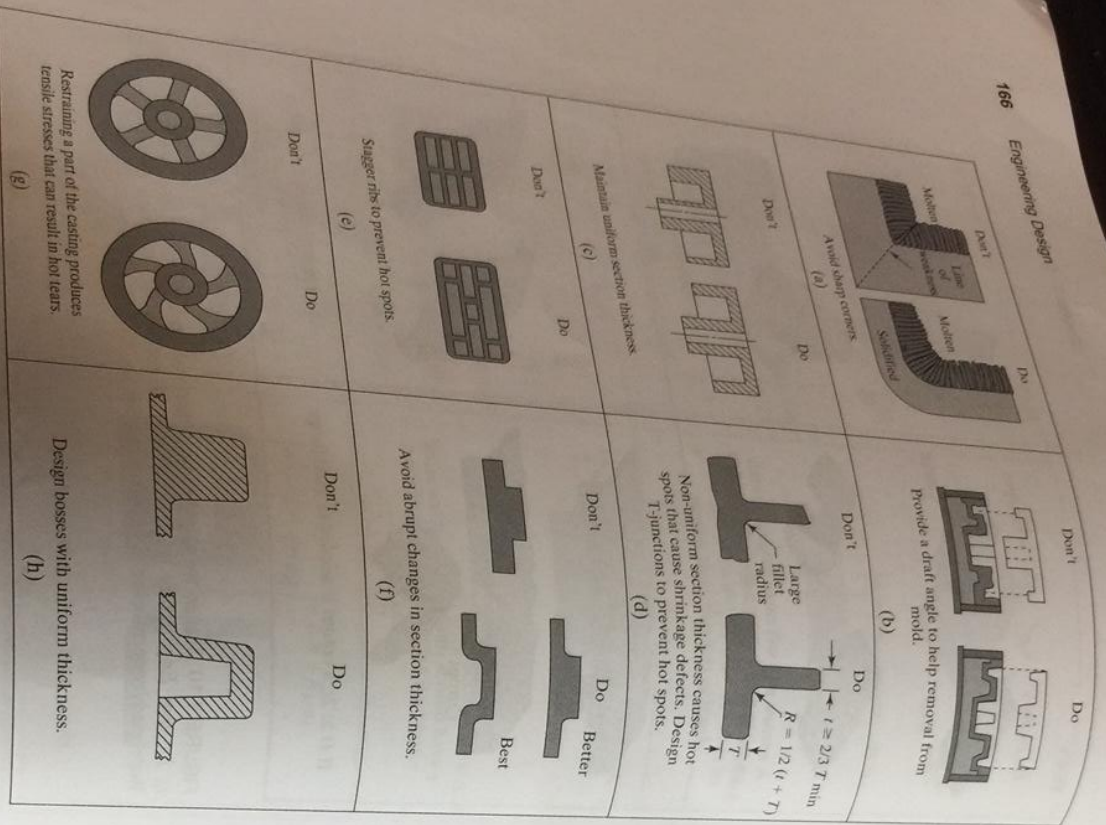


FIGURE 7.11 Design for manufacture of cast parts. (*Product Design: Techniques in Reverse Engineering and New Product Development*, by Otto Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

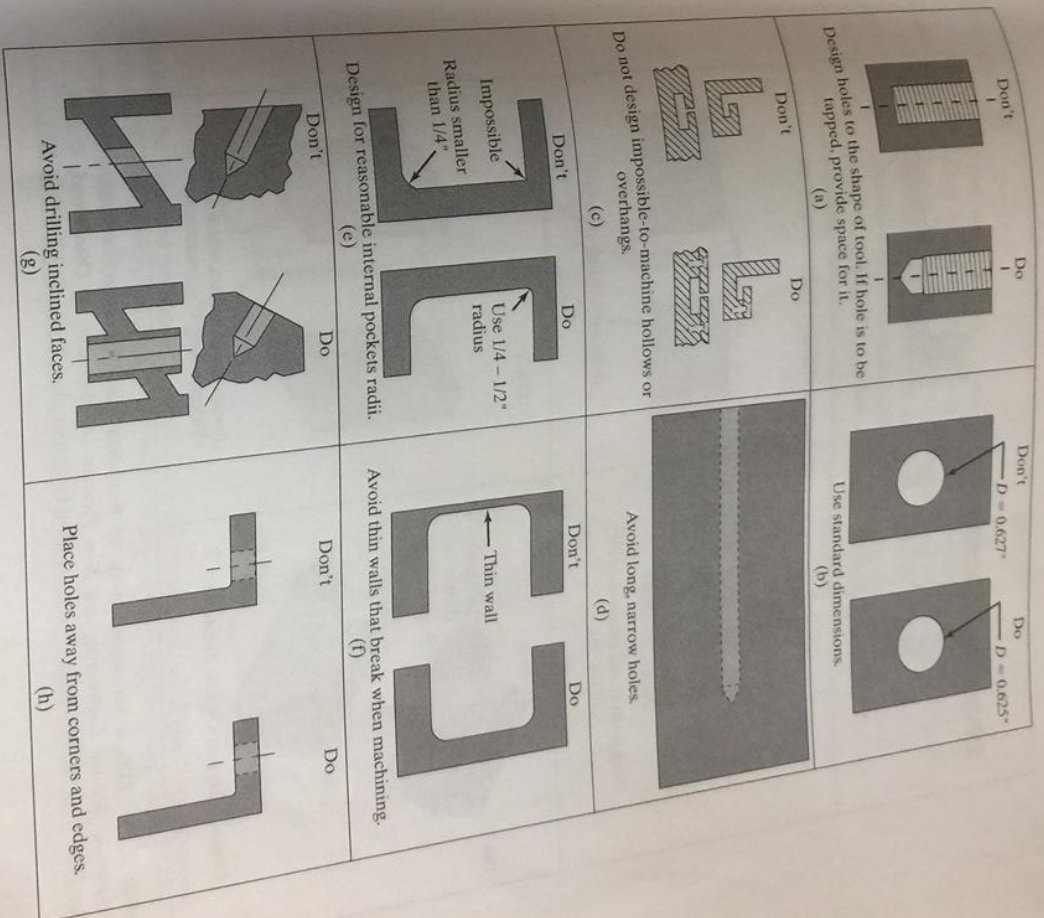


FIGURE 7.12 Design for manufacture of machined parts. (*Product Design: Techniques in Reverse Engineering and New Product Development*, by Otto Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

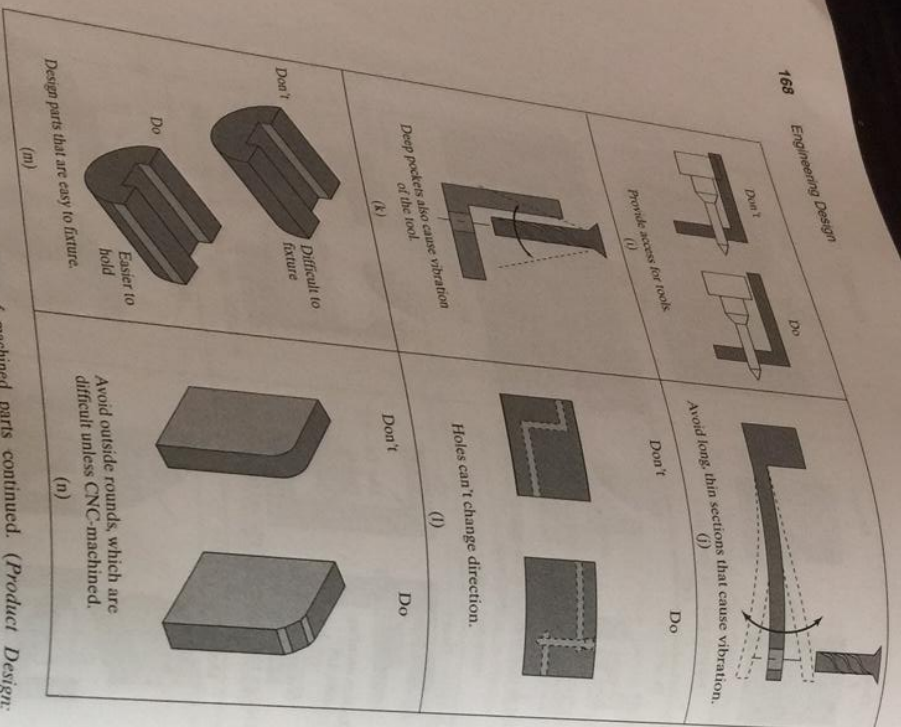


FIGURE 7.13 Design for manufacture of machined parts continued. (*Product Design: Techniques in Reverse Engineering and New Product Development*, by Otto/Wood, © 2001, Prentice-Hall. Reprinted by permission of Pearson Education, Inc., Upper Saddle River, NJ.)

Example Analyze the four contiguous part configuration sketches of the sponge holder shown in Figure 7.6 and develop two refined configurations for evaluation.

The sponge holder design was revised, as shown in Figure 7.14, after careful consideration of the design for function, assembly, and manufacture guidelines. We see that the bottom has been removed and the front has been opened for air circulation, easy cleaning, and sponge egress, thereby improving overall function. Sides have been tapered to improve packaging nesting and reduce material usage. Walls have been made uniformly thin and fillets have been added to improve molding.

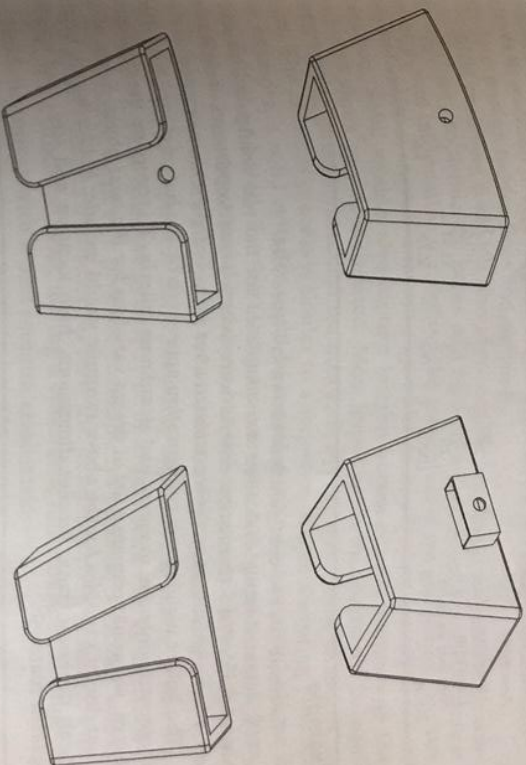


FIGURE 7.14 Back and front views of two sponge holder configurations. The left configuration shows that the suction cup pin penetrates the body of the holder, possibly inhibiting sponge egress. The right configuration eliminates this problem.

7.4 EVALUATING CONFIGURATION ALTERNATIVES

As we finish making refinements to our configurations, we may need to reduce the number of alternatives. Using Pugh's method or the weighted-rating method, we select evaluation criteria for function, manufacture, and assembly. As a team, we also establish importance weights, and then rate our alternatives.

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TABLE 7.4 Weighted Rating Evaluation of Spoke Holder Configurations

Criteria	Importance weight	Configuration			
		With hole	With bracket	With hole	With bracket
		Rating	Wt. Rating	Rating	Wt. Rating
Function	15	3	0.45	3	0.45
Drain quickly	10	3	0.30	3	0.30
Does quickly	10	3	0.30	3	0.30
Spoke clear	10	2	0.20	4	0.80
Spoke inserts easily	10	2	0.20	2	0.20
Material usage	15	3	0.45	3	0.45
Material costs	10	3	0.30	3	0.30
Tooling costs	10	3	0.30	3	0.30
Processing costs	10	3	0.30	3	0.30
Assembly	10	3	0.30	na	2.95
Handling	10	na	2.75	na	2.95
Insertion	100%	na	2.75	na	2.95
Number of parts	100%	na	2.75	na	2.95
Total					

Example

Evaluate the two configurations shown in Figure 7.14. A weighted-rating evaluation was completed for the two configurations and is shown in Table 7.4. Evaluation criteria were selected and importance weights estimated. The weighted ratings were calculated after each configuration was rated.

The holder with bracket configuration appears to be the better choice for further development mainly due to egress and cleanliness criteria. To complete the configuration design, we would prepare a list of design variables for the next phase. Some of the important design variables for the above example would include: holder width, height, depth, wall thickness, radii, bracket dimensions, hole location and size, and specific thermoplastic material.

7.5 COMPUTER-AIDED DESIGN

Configuration design activities revolve around product architecture and part configuration. We make significant use of hand-drawn sketches to work out and improve our ideas and communicate them to our team members. We use spreadsheets for checklists and weighted-rating evaluations. Similarly, we can make use of solid modeling packages. Note, however, that at this phase we are not trying to produce a set of working drawings such as detail part drawings or assembly drawings with bills of materials. Rather, our goal is to generate, analyze, and evaluate alternative configurations, *not document* our design efforts.

Computer-aided design (CAD) hardware and software can be used to assist in the creation and development of a design. The simplest software

packages represent parts as *two-dimensional models*. Similar to the classic package, board, orthographic views of a three-dimensional object are created using points, lines, circles, and arcs. With this type of software three-dimensional ratings cannot be produced. A major disadvantage of two-dimensional systems is that if we change the length of a side we have to revise any lines, arcs, or circles attached to the side. Wireframe packages model a part using a three-dimensional database to store and manipulate the edges of the part. Once the wireframe views are essentially automated, however, generation of the orthographic views is essentially automated. In some cases, complex wireframe parts can be confusing because of all the "hidden" lines that are exposed. *Surface modeling* packages define the faces of a part in terms of flat and curved surfaces in addition to representing edges. This has the advantage of automatically determining hidden lines and surfaces.

Solid modeling packages also represent the wireframe and surface geometry necessary to define the edges and faces of a part. A solid model contains topographical information that defines the association of the geometrical entities such as faces that share a common edge which makes filleting a straightforward operation. Solid models also capture interior versus exterior features so that sectioned views can be automated. Most important is that parts can be configured having three-dimensional features such as ribs, slots, holes, chamfers, and fillets. The creation process is similar to a sequence of manufacturing processes. For example, a solid cylinder could be modeled with a concentric hole, as if it were drilled or bored. Further, these features are not stored as arcs, lines, or points, as in two-dimensional packages, but rather as solid objects. Representing part information as a virtual solid has a number of advantages:

Design Intent Is Captured. Solid modeling captures the features the designer intended. That is, as the geometry of each part is inevitably modified during the iterative steps of a product development process, each part will respond to these changes according to how the designers have planned. For example, if a cylindrical boss, having a concentric hole through it, is moved, then the hole moves with it and remains concentric.

Feature-Based Modeling. The creation of parts is simplified by automating the creation of standard manufacturing features such as bosses, cutouts, holes, ribs, fillets, chamfers, and draft. Rather than drawing an arc connected by lines, for example, the solid-modeling designer selects the "fillet" feature to automatically generate the arc to the desired radius.

Constraint-Based. Control of part geometry is maintained using constraints. For example, selected surfaces can be constrained to be parallel or perpendicular, holes can be made concentric, rounds can be made tangent, or the location of a slot can be fixed.

Parametric. Solid modeling is also parametric, in that the parameters of a model may be modified to change the geometry of that model. Instead of fixing the diameter of a hole as 5 inches, solid modeling uses a variable for the diameter that can be modified as the design evolves. Therefore, when a dimension value is changed, the geometry of the part is automatically updated.

[illegible][illegible]

this same with respect to other parts of the design process, motion with respect to other parts of the design process.

Downstream Benefits: A major advantage of solid modeling is that the data generated by the model can be exported to other downstream software packages. These packages include finite element analysis (FEA) software, used to predict product performance and thermal and support conditions to which the forces, pressures, and temperature are subjected can be simulated and the individual parts or an entire assembly can be modeled and improved upon to avoid stress resultant patterns of stress, deformation, and undesirable temperatures. In addition, these results part designs can be revised and improved upon to avoid stress concentrations, large deformations, and undesirable temperatures. In addition, the same database can be exported to a rapid prototyping system to produce physical prototypes, often within a matter of hours.

The prototypes serve to validate fit, form, and function. Once the design is finalized, the solid-modeling software can be used to generate standard engineering drawings. Computer-aided manufacturing (CAM) software can also be downloaded to a computer numerical control (CNC) machine tool to automatically manufacture production parts to within specified tolerances.

7.6 SUMMARY

- Product configuration or architecture design determines the type and number of components, their arrangement, and their relative dimensions.
- Part configuration design determines the type and number of geometric features, their arrangement, and their relative dimensions.
- Standard part configuration design involves determining part type and relative dimensions.
- Part features include: walls, ribs, projections, fillets, bosses, rounds, cylinders, tubes, cubes, spheres, holes, slots, notches, chamfers, and grooves.
- Configuration requirements sketches can be used to develop alternative part configurations.
- Configuration analysis includes considerations of function, assembly and manufacture.

- Alternative configurations may undergo significant revision during successive iterations. Solid-modeling CAD systems can be useful during configuration design as well downstream in parametric design and/or manufacturing.

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KEY TERMS

- Arrangement (location/orientation)
- Chunk
- Cluster
- Computer-aided design (CAD)
- Computer-aided engineering (CAE)
- Configuration design
- Configuration requirements sketch
- Connectivity
- Contiguous configurations
- Design for assembly (DFA)
- Design for manufacture (DFM)
- Features
- Function sharing

EXERCISES

Self-Test. Write the letter of the choice that best answers the question.

1. d When configuring a product, we need to decide: a. geometric features
b. component types c. design variables d. a, b, and c
2. d When configuring a special-purpose part, our goal is to determine: a. type
and number of features b. their arrangement or connectivity c. their
dimensions relative to each other d. a, b, and c

CHAPTER 8

Parametric Design

LEARNING OBJECTIVES

When you have completed this chapter you will be able to

- Describe design phase information inputs/outputs
- Specify solution evaluation parameters
- Identify and characterize design variables
- Establish explicit and implicit constraints
- Select and use methods to analyze alternative designs
- Develop parametric spreadsheets to refine designs
- Characterize and assess overall customer satisfaction
- Explain design for robustness

8.1 INTRODUCTION

Parametric design includes a number of decision-making processes, just like the other phases of design. The processes use information (input) from prior phases to arrive at logical decisions (output). What makes parametric design special and particularly challenging is that we will employ analytical and experimental methods to predict and evaluate the behavior of each of the design candidates to make these decisions.

Let's examine the types of input and output information that we will process as we make our decisions in the various design phases.

8.1.1 Information Flow in the Design Phases

As we recall the phases of design as shown in Figure 8.1, information about the customers' needs is processed in the formulation phase usually resulting in a list of customer requirements/product functions, the importance of each, a list of engineering characteristics that quantitatively describe how well the functions are performed, a detailed engineering design specification, and ideally, a House of Quality diagram.

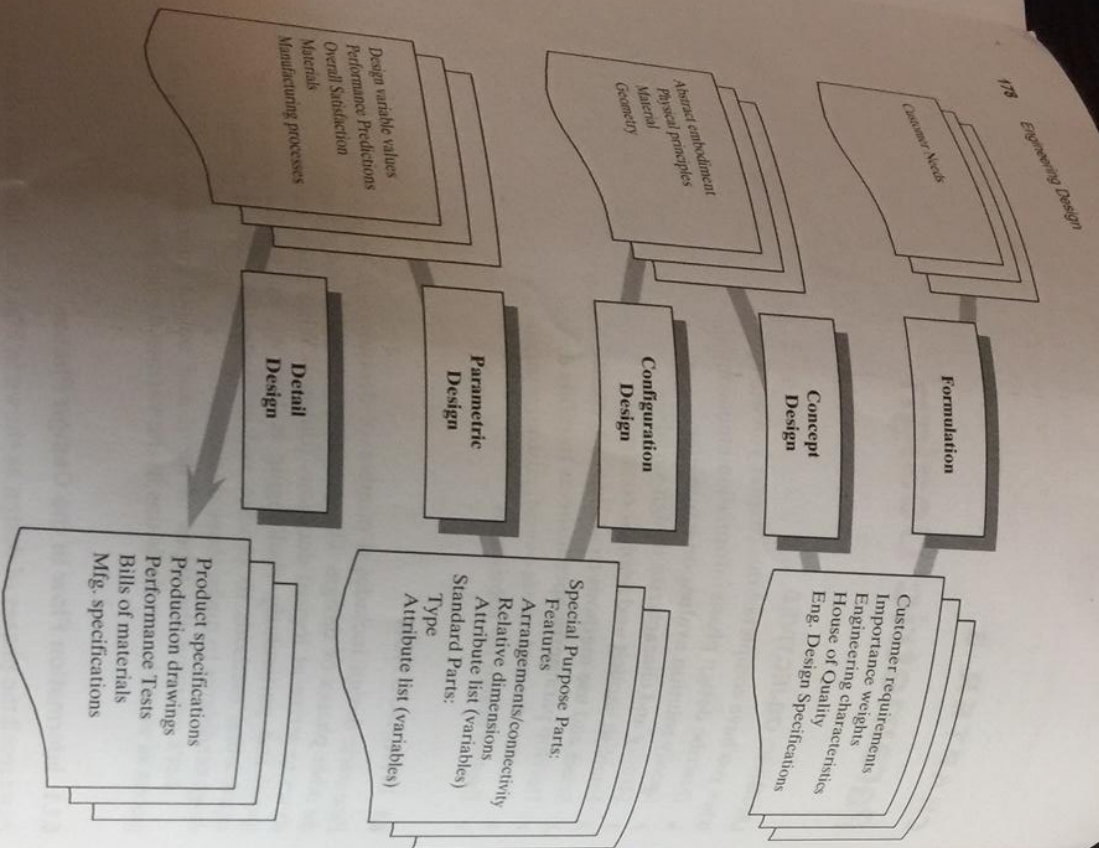


FIGURE 8.1 Design phase information input/output.

We use that information as input to the concept design phase, wherein we make decisions about the physical principles, abstract embodiment, primary manufacturing processes, and material classes.

Similarly, we use that information as input to the configuration design phase, wherein we make decisions about the product's architecture to decide which parts should be standard or special purpose. Then we determine special-purpose parts geometric features, their arrangement and/or connectivity, and their relative dimensions. And we establish a list of attributes or variables that we will need values for. And for standard parts, we select the specific type and we will need values for.

In the **parametric design** phase, we determine the values of those attributes, typically called **design variables**. These usually relate to specific attributes, lengths, radii, diameters, material types, and manufacturing process sizes. Each set of design variable values is called a design candidate. We predict the performance of alternative candidates using analytical methods and/or experimental methods. If the performance of the candidates satisfies all the design constraints, we call them **feasible designs**. Then we evaluate the feasible designs to determine which candidate is the best.

Finally, during the detail design phase, we complete the remaining decisions, resulting in comprehensive product specifications, drawings, manufacturing specifications, performance tests, and bills of materials. Next, we examine the parametric design of a simple flange bolt.

8.1.2 Parametric Design: Pipe Flange Bolt Example

At a petrochemical plant, eight bolts will be used to fasten a cooling water pipe to the inlet flange of a high-pressure chemical reactor vessel. The piping system is subjected to an operating pressure of 1,000 psi. The flanges will be subjected to temperatures up to 250°F. Each bolt will be required to withstand a 4,000 pound design overload, which is four times the typical operating load of 1,000 pounds. The company has suggested a standard hex-head bolt, shown in Figure 8.2. Some of the design variables for bolts include: bolt diameter d , in Figure 8.2, number of threads per inch, and type of material. Assume overall length L , number of threads per inch, and type of material. Assume that for this example, we have been asked to determine the bolt diameter only.

The principal function of a bolt is to fasten parts together so that they can be disassembled later for maintenance or repair. As the bolt is tightened, a tensile force develops over its grip length. The greater the force, the better the clamping action. Also, since the head of the bolt consequently develops higher friction forces, there is an improved resistance to loosening.

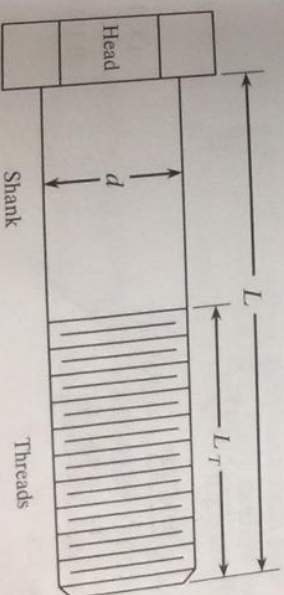


FIGURE 8.2 Hex-head bolt.

However, a bolt can fail to function in a number of ways, including (1) the bolt head can twist off during tightening, (2) the bolt threads can strip off, and (3) the bolt can experience excessive tightening or excessive operating loads that it permanently elongates or pulls apart.

Let's consider only the tensile force without a permanent measurable elongation or "set" is called the proof load F_p , and is directly proportional to cross-section area A of the bolt and the material's proof strength S_p , by the relation:

$$F_p = A S_p \quad (8.1)$$

The Society of Automotive Engineers (SAE) has categorized proof strengths of bolt materials as Grades 1-9, with strengths ranging from 33,000 psi to 120,000 psi, respectively. Assume that we select a Grade 5 material, whose proof strength is 85,000 psi.

Therefore, for this design problem, we need to design the bolt strong enough to have a proof load that is greater than the 4,000 (lbf) design load. This is a constraint for this problem. Consequently, we need to find an area and, therefore, diameter such that:

$$F_p \geq 4000 \text{ lbs} \quad (8.2)$$

By substituting equation (8.1) into the constraint equation (8.2), we obtain:

$$A S_p \geq 4000 \text{ lbs} \quad (8.3)$$

We find that the proof-load constraint can be satisfied if the area:

$$A \geq \frac{4,000}{S_p} \quad (8.4)$$

$$A \geq \frac{4,000 \text{ (lbs)}}{85,000 \text{ (lbs/in}^2\text{)}} \quad (8.5)$$

$$A \geq 0.047 \text{ in}^2 \quad (8.6)$$

and since cross-sectional area is related to diameter d as:

$$A = \frac{\pi d^2}{4} \quad (8.7)$$

$$\text{substituting, we find that } \frac{\pi d^2}{4} \geq 0.047 \text{ in}^2 \quad (8.8)$$

$$\text{and further, that } d^2 \geq \frac{0.047(4)}{\pi} = 0.0598 \text{ in}^2 \quad (8.9)$$

$$d \geq 0.245 \text{ in.} \quad (8.10)$$

The calculated diameter is then rounded up to the nominal size 0.25 in., so that a standard size could be purchased.

As shown in Figure 8.3, as we increase the diameter, the calculated proof load increases exponentially. The required proof load is constrained to be equal to or greater than 4,000 (lbs.) and is shown as a horizontal line. The minimum feasible diameter is shown as the dashed line at 0.245 (in.) and, therefore, satisfies the constraint. Selecting a diameter larger than 0.245 (in.) would improve the safety and reliability of the design and could be considered in the final design.

This parametric design problem is rather simple, in that we have only one governing constraint (designed proof load must be greater than 4,000 (lbs.)) and one algebraic relation to substitute for the area of a circular cross section. Therefore, we can directly solve for the diameter that satisfies the constraint. There is no need to iterate. Some engineers call this type of problem **inverse analysis**, because we can rearrange an analysis equation(s) to find the unknown value. Analytical relations are often derived from experiment as known value. $\text{function} = f(\text{form})$. By inverting the analysis equation we find $\text{form} = f^{-1}(\text{function})$.

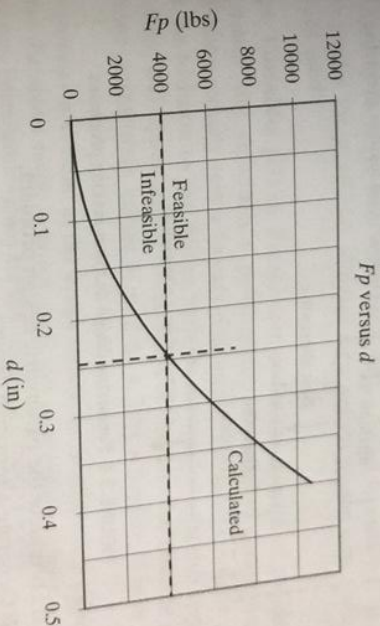


FIGURE 8.3 Flange bolt proof load versus diameter.

The parametric design of most mechanical parts is much more difficult. Let's examine the overall process to figure out how we can more systematically, and thereby complete better designs with less effort.

8.2 STEPS IN SYSTEMATIC PARAMETRIC DESIGN

Systematic parametric design has five major steps, as shown in Figure 8.4:

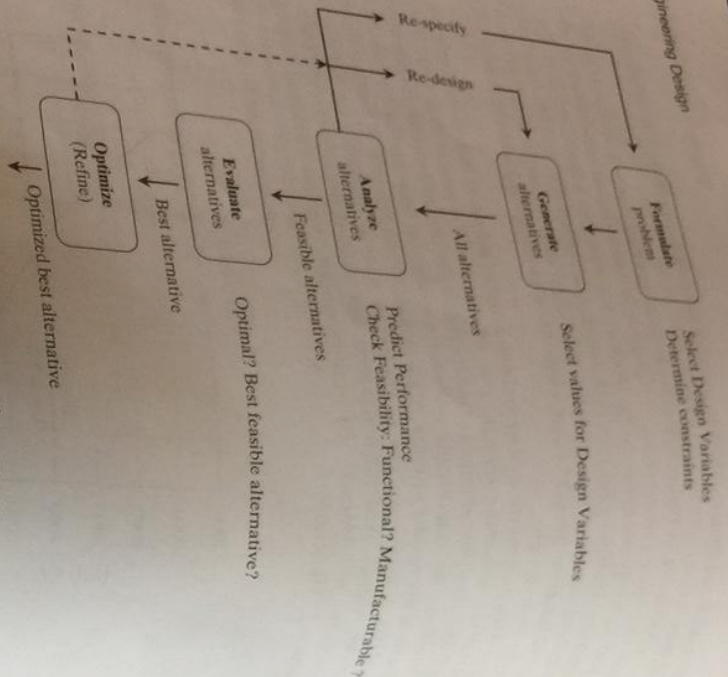


FIGURE 8.4 Parametric design decision-making processes.

- Step 1:** Formulate the parametric design problem.
- Step 2:** Generate alternative designs.
- Step 3:** Analyze/predict the performance of the alternatives.
- Step 4:** Evaluate the performance of each alternative.
- Step 5:** Optimize/refine.

Step 1. Formulate the problem. Practicing engineers often comment that they spend 40–50 percent of their time gathering appropriate data, clarifying important details, considering different analytical and experimental analyses, and then planning how they will complete a project. All these activities relate to problem formulation. As we start a parametric design problem, we need to familiarize ourselves with the problem parameters and also plan ways complete the design.

Solution Evaluation Parameters (SEPs). By reviewing the House of Quality chart and the engineering design specifications, we reaffirm what functions the customer wants the product to perform. We then select engineering characteristics to measure the predicted performance of the functions. Solution evaluation parameters are the engineering characteristics that are selected to determine how well a candidate design “solves” the problem (Dixon and Poll, 1995).

Solution evaluation parameters depend upon the product and part being designed, but often include: cost, weight, speed, efficiency, safety, and reliability. We also denote parameter symbols, units (of measurement), and any lower and/or upper limits of the parameter. As mentioned in Chapter 3, we can also describe the customer’s satisfaction with respect to each solution evaluation parameter, as a satisfaction curve. Finally, we agree on analytical or experimental methods to determine their values.

In the flange bolt example, the customer wants the bolt to clamp the pipe flange to the pressure vessel flange. The clamping force can be measured by the proof load and is, therefore, an example of a solution evaluation parameter. The higher it is, the better the clamping force, and the more satisfied the customer. We selected F_p as its symbol and pounds-force as the units. A lower limit, 4,000 (lbf), is also established. The bolt proof load is constrained to be larger than 4,000 (lbf). An analytical formula will be used to predict the proof load, although experimental tension tests could have been used.

Design Variables (DVs). Parameters under the control of the designer, which influence the candidate’s performance, are called design variables. Design variables usually relate to part dimensions, tolerances, and/or material properties. In addition to the design variable names, we establish appropriate symbols, units, and upper and/or lower limits or bounds. For discrete variables we determine permissible values (e.g., 2 by 4, 2 by 6 lumber, $\frac{1}{2}$ inch diameter bolts).

The diameter is selected as the design variable in the flange bolt example. Larger diameters result in larger areas and consequently, improved performance. Permissible values are identified using handbook data.

Problem Definition Parameters (PDPs). Parameters that describe specific conditions of use, such as operating conditions are called problem definition parameters. In addition to their names, we establish appropriate symbols, units, and values.

The operating pressure of 1,000 psi and the limiting temperature of 250° F are identified as PDPs for the flange bolt example.

Preliminary Plan for Solving the Problem. For small design problems, we often jump right in and start calculating things. For larger, more involved problems, we need to make a preliminary plan based on considerations including:

1. Do we have analytical models/formulas for our problem?
2. Are the assumptions used in our models the same as our problem?

3. Will we need to perform pilot scale or bench-top experiments to validate our analytical formulas?
 4. How much time and money do we have to "solve" the problem?
 5. Are "ballpark" computations required, or do we need more thorough and precise calculations?
 6. Do we have knowledge about acceptable industry standards?
 7. Will manufacturing specifications need to be generated from our analyses?
 8. Do we understand the customers' function requirements versus satisfaction well enough?
 9. Are we sufficiently qualified or competent to complete the required design?
- For large design problems, design teams will consider the questions above and will prepare a design project proposal for upper management to review and approve. Design project proposals include items such as a background section, goals or mission of the project, a scope of work (of the work to be performed), schedule, and budget. Regardless of the size of your project, however, it is always a good idea to prepare an outline of what you are going to do and when and how you are going to do it.

Recognizing the simple nature of the flange bolt calculations, we went straight to making a few calculations.

Step 2. Generate alternative designs. We select different values for the design variables to generate different candidate designs. These values can come from our own experience, from our company's experience, or from industry standards. Sometimes we need to make educated guesses.

Step 3. Analyze the alternative designs. We predict the performance of each candidate design using analytical and/or experimental methods.

Analytical Methods. Formulas from physics, mathematics, and the engineering sciences are most often used. Sometimes, advanced computer-aided design packages, such as finite element analysis, computational fluid dynamics, and motion simulation, are used.

Experimental Methods. Oftentimes, the complexity of the design is beyond the accuracy or assumptions of our analytical models. In these cases, we can build scale models and/or full-scale models of the product or critical parts of the product, and test their performance. We can use wind tunnels, for example, to analyze the performance of complicated wing geometries, or check control surfaces.

The performance of each design candidate is checked so that every performance constraint is satisfied. These designs are called feasible designs. If the constraints are violated, we reiterate back to generating another alternative and then analyzing it.

As we substitute new values for the design variables into the system of analysis equations we become familiar with how each design variable influences part or product performance. Using this familiarity, we can sometimes generate new values that satisfy the constraints. Dixon and Poli (1995) call this

"physical reasoning." In other words, by becoming familiar with the causes and effects, we can logically reason the "physics" of the problem.

If no feasible design candidates exist, we select new values for the design variables, and thereby generate new design candidates. These are subsequently analyzed for feasibility. This is the solid "redesign" iteration loop shown in Figure 8.4. If we cannot find any feasible candidates, we might ask whether we have set our design specifications too restrictively. Perhaps one or more constraints could be relaxed. If so, we are "respecting" as shown in Figure 8.4.

The formula for the area of a cylindrical cross section (8.7) and the formula relating the proof load of a bolt to its area and strength (8.1) were used in the bolt example. The performance constraint considered in the bolt example was that the proof load be larger than 4,000 (lbs.). Since the formulae were straight forward, we were able to juggle the equations into a sequence that did not require iteration. More complicated problems will not be "solvable" by equation juggling.

Step 4. Evaluate the results of the analyses. The feasible designs are evaluated to determine the best design. Usually one or more criteria are identified in the formulation phase and used to determine the "best" feasible design alternative.

The proof load was established during formulation as a rough measure of customer satisfaction. We assume that the higher the proof load, the more satisfied our customer would be. That would mean that a large bolt should be chosen as the "best" design. This, however, ignores other aspects of real design problems such as weight and cost limitations. Unfortunately, since we do not know how the customer feels about these issues, we can only surmise that he might be satisfied with the smallest bolt that meets the force constraint. As we shall see in the next section, we should always try to ascertain customer satisfaction with respect to each solution evaluation parameter.

Step 5. Optimize/refine. Optimal design methods automatically regenerate new values of the design variables to improve expected performance and satisfaction (Arora, 1989; Papalambros and Wilde, 1989; Rao, 1984; Reklaitis et al., 1983; Siddall, 1982; Vanderplaats, 1984). For single-attribute optimization, a single criterion is chosen, such as minimizing the weight of a part. The criterion is a function of the design variables, and is called the **objective function**. Design variable values are usually constrained to some upper and lower limits. The predicted performance of a part is also related to the design variables. For example, the strength of a bolt is a function of its diameter. Design variable functions and limits are called **constraints**. Equation (8.2), for example, is an inequality constraint. Excel's Solver feature can be used to optimize many typical design problems found in mechanical engineering. This is shown as the dashed regenerate loop in Figure 8.4.

A more detailed example using this systematic procedure is presented in the next section.

8.3 SYSTEMATIC PARAMETRIC DESIGN: BELT-AND-PULLEY EXAMPLE

Let's apply the systematic procedure presented in the last section using a more detailed example of a belt-and-pulley transmission.

8.3.1 Design Problem Formulation

A $\frac{1}{2}$ -hp electric motor, running at 1,800 rpm, will be used to drive a grinding wheel operating at 600 rpm. A flat belt-and-pulley drive system configuration has been selected, as shown in Figure 8.5. The design team has also determined that:

- the drive motor will have a 2-in. diameter pulley mounted,
- candidate designs should be able to transmit the full horsepower,
- the customer desires a compact system design,
- the drive pulley will slip first, before the driven pulley,
- the purchasing department has located a vendor that can provide a flat belt that can withstand a maximum 35-pound tensile load,
- the coefficient of friction between the belt and pulley is 0.3,
- other design engineers in your group will design the mountings, bearings, and protective equipment, therefore,
- and protective design efforts should focus on the distance between centers and the diameter of the driven-pulley.

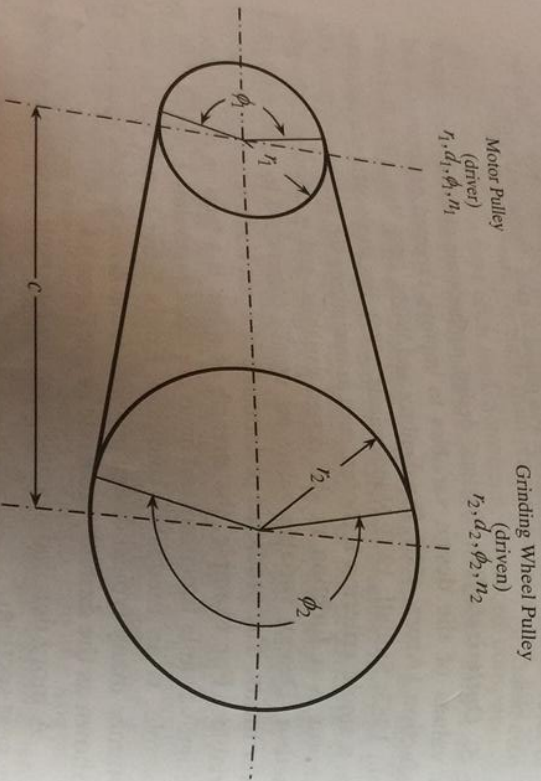


FIGURE 8.5 Belt-and-pulley-drive system for motor and grinding wheel.

Solution Evaluation Parameters (SEPs) The principal function of the pulley is to transform the power of the motor from a high speed to low speed. In the transformation, the smaller motor torque is converted to a larger torque, according to the conservation of energy law. Also, the belt-and-pulley system would fail to perform its principal function if the belt slipped or if the belt broke owing to excessive tension. Finally, the customer would be more satisfied with a compact design.

Since we know that the tension force in the belt is limited by the amount of friction between the belt on the driver pulley, up to the point of impending slip, we could determine the torque that the belt can deliver to the pulley, T_b , and compare it with the maximum torque, T_m , that the motor can supply. Also, we should calculate the maximum belt tension, F_1 , to make sure that it does not exceed the 35 (lbs.) limit.

We can summarize the solution evaluation parameters in Table 8.1.

TABLE 8.1 Solution Evaluation Parameters for Belt-Pulley System

Parameter	Symbol	Units	Lower Limit	Upper Limit
1 belt torque	T_b	lb-in	T_m	—
2 belt tension	F_1	lbs	—	35
3 center distance	c	in.	small	—

Design Variables (DVs) The value of the center distance, c , directly affects the compactness of the design and is to be determined by the designer. Also, as the center distance is increased, more of the belt wraps around the pulley (i.e., is in contact with the surface of the pulley), increasing the ability of the belt to grip the pulley and thereby satisfy the torque requirements of the motor. The design variables are summarized in Table 8.2.

TABLE 8.2 Design Variables for Belt-Pulley System

Design Variable	Symbol	Units	Lower Limit	Upper Limit
1 center distance	c	in.	small	—
2 driven pulley diameter	d_2	in.	—	—

Problem Definition Parameters Studying the design problem data, we find a number of “givens” that define design problem conditions such as the friction coefficient, belt strength, motor power, and motor pulley diameter. Therefore, we identify these as the problem definition parameters in Table 8.3.

TABLE 8.3 Problem Definition Parameters for Belt-Pulley System				
Parameter	Symbol	Units	Lower Limit	Upper Limit
friction coefficient	f	none	0.3	0.3
belt strength	F_{max}	lbs	—	30
motor power	$\dot{W}$	hp	—	1/2
motor pulley diameter	d_1	in.	2	2

Plan for Solving the Design Problem Using analytical relations from physics and mathematics we can use a hand calculator or develop a spreadsheet to calculate the engineering characteristics, including:

1. grinding wheel pulley speed, n ;
 2. angle of wrap as a function of the center distance, c ;
 3. belt torque, T_b ;
 4. maximum belt tension, F_1 ;
 5. slack-side belt tension, F_2 , and
 6. initial tension (before torque is applied), F_i .
- Then we will check that the constraints are not violated. Specifically, we will make sure that the belt will deliver the full motor torque to the grinding-wheel pulley and that the belt tension does not exceed the belt strength limit.

8.3.2 Generating and Analyzing

Following our plan, we need to develop an analytical means to predict the behavior of the system. We can model the behavior of the system using relations from physics and mathematics and develop a system of equations to analyze the performance of each candidate design as we substitute different values for the design variables.

For example, we know that a motor will deliver power $\dot{W}$ (hp), to a pulley rotating at n rpm when producing a torque T_m lb. ft. according to equation (8.11):

$$\dot{W} = \frac{T_m n}{5252} \quad (\text{hp}) \quad (8.11)$$

If the belt is not permitted to slip on the pulleys, the pulley speeds are related to the ratio of the pulley diameters.

$$n_2/n_1 = d_1/d_2 = r_1/r_2 \quad (8.12)$$

We can determine the angle of wrap ϕ_1 using basic geometric relations.

$$\phi_1 = \pi - 2 \sin^{-1} \left(\frac{d_2 - d_1}{2c} \right) \quad (8.13)$$

Similarly, we know that a belt, with coefficient of friction f that is in contact with the pulley for an angle of wrap ϕ_1 , has a maximum belt tension F_1 on the taut side of the pulley, and is related to the belt tension F_2 on the slack side of the pulley according to:

$$\frac{F_1}{F_2} \leq e^{f\phi} \quad (8.14)$$

Upon sketching the free-body diagram, shown in Figure 8.6, we see that for static equilibrium, we can sum the moments about the bearing B to obtain the torque T_m delivered by the belt to the driver pulley of radius r_1 , as:

$$T_b = (F_1 - F_2)r_1 \quad (8.15)$$

As a point of interest, we can show how the angle of wrap affects the torque capacity of the belt at impending slip, by substituting equation (8.14) into (8.15). By examining the resulting equation (8.16), we see that as the angle of wrap increases and the torque increases.

$$T_b = (F_1 - F_2)r_1 = \left(F_1 - \frac{F_1}{e^{f\phi}} \right) r_1 = F_1 \left(1 - \frac{1}{e^{f\phi}} \right) r_1 \quad (8.16)$$

The maximum torque delivered by the motor, T_m can be determined by rearranging equation (8.10) as:

$$T_m = \frac{\dot{W} 5252}{n} = \frac{0.5(5252)}{1800} = 1.46 \text{ (ft lbf)} = 17.52 \text{ (in. lbf)} \quad (8.17)$$

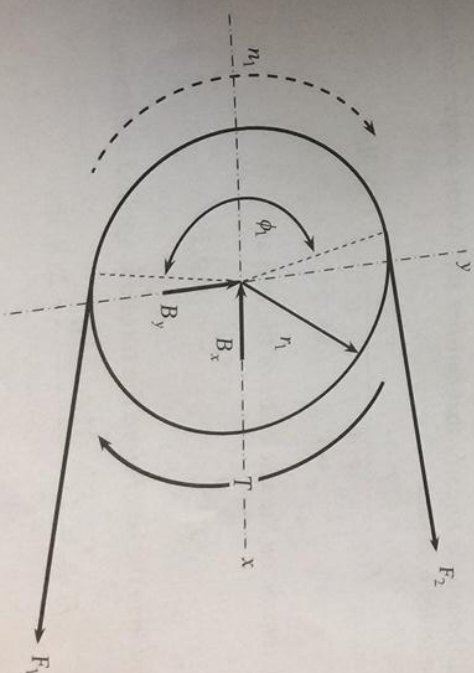


FIGURE 8.6 Free-body diagram of motor pulley.

The belt must be able to provide a torque equal to or greater than 17.52 (in·lb) or else it will slip. otherwise it will slip. we can predict the driven-pulley speed, n_2 , as a function of the design variable, diameter d_2 .

Using equation (8.12) $d_1 = 1800 \frac{2}{d_2}$ (rpm)

$n_2 = n_1 \frac{d_1}{d_2}$ (8.18)

Note that equation (8.18) is an equality constraint, having one unknown design variable, d_2 . A feasible candidate design must satisfy this constraint. Since the grinding-wheel speed has been specified as 600 rpm, we can obtain a feasible value of the driven-pulley diameter by rearranging equation (8.18).

$d_2 = d_1 \frac{n_1}{n_2} = 2 \left(\frac{1800}{600} \right) = 6$ (in.) (8.19)

Therefore, the only feasible value of the design variable d_2 is 2 in.

Generating an Initial Value of the Design Variable c Since the customer will be more satisfied with a compact design, we would like the two pulleys can be the pulley centers, c to be small. The closest that the two pulleys can be is when their radii are almost touching, theoretically speaking, or

$d_1 + d_2 = \frac{6 + 2}{2} = 4$ in. (8.20)

Now we can use equation (8.13) to find the angle of wrap on the driving pulley.

$$\phi_1 = \pi - 2 \sin^{-1} \left(\frac{6-2}{2(4)} \right) = 2.09 \text{ rad} = 120 \text{ deg} \quad (8.21)$$

To find the tensile force F_1 that satisfies the motor torque constraint we use equation (8.16) to obtain

$$F_1 = \frac{T}{\left(1 - \frac{1}{e^{\phi_1 \mu}}\right)} = \frac{17.52}{\left(1 - \frac{1}{e^{0.3(2.09)}}\right)} = 37.5 \text{ lbs} \quad (8.22)$$

We find the tension on the slack side of the belt as

$$F_2 = \frac{F_1}{e^{\phi_1 \mu}} = \frac{37.5}{e^{0.3(2.09)}} = 20.0 \text{ lbs} \quad (8.23)$$

The initial tension in the belt before the torque is applied is obtained as

$$F_i = \frac{F_1 + F_2}{2} = \frac{37.5 + 20.0}{2} = 28.8 \text{ lbs} \quad (8.24)$$

We check the belt torque using equation (8.15).

$$T_b = (F_1 - F_2)r_1 = (37.5 - 20.0)1 = 17.5 \text{ lb} \cdot \text{in} \quad (8.25)$$

We note that to satisfy the torque constraint, a 37.5-lb. tension will be necessary for F_1 . But that level of tension exceeds the belt strength constraint of 35 lbs. Therefore, a center distance of 4 in. is an infeasible value. It causes a violation of a constraint and will need to be increased (to increase the angle of wrap, and consequently reduce the tension in the belt).

To reduce the effort in computing the expected system performance as a function of center distance, c , we can develop a spreadsheet as shown in Table 8.4.

TABLE 8.4 Flat Belt & Pulley Design

Problem Definition Parameters		Symbol	Units	Value			
friction coefficient	f		none	0.3			
belt strength	$F_{\max}$		lbs	35			
motor power	W		hp	0.5			
motor speed	n_1		rpm	1800			
motor pulley diameter	D_1		inches	2			
Design Variables		Symbol	Units	Lower	Value	Upper	
driven pulley diameter	d_2		inches	—	6	—	
center distance	c		inches	4.0	10	12	
Performance Calculations		Symbol	Units	Value	Type	Value	Constraint Condition
Eng. Characteristic							
motor torque	T_m		lb-in	17.51			
grinding wheel speed	n_2		rpm	600	=	600	Satisfied
angle of wrap	ϕ_1		degrees	120.0			
belt tension-taut	F_1		lbs	37.5	≤	35	Unsatisfied
belt tension-slack	F_2		lbs	20.0			
initial belt tension	F_i		lbs	28.8			
belt torque	T_b		lb-in	17.51	≥	17.51	Satisfied

Redesigning: Finding Feasible Values Since the belt tension is constrained to a maximum of 35 lbs., the initial value chosen for c is infeasible. We therefore increase the center distance and recalculate the belt tensions. Using the spreadsheet, we obtain the following values for belt tension, F_1 in Table 8.5.

TABLE 8.5 Belt Tension for Alternative Center Distances

Center distance c (in.)	Belt Tension F_t (lbs)
37.5	35.0
35.0	33.5
33.5	32.0
32.0	31.2
4.96 (Goalseek)	
4	
6	
8	
10	

Using Excel's Goalseek feature we find when $c \geq 4.96$ in., the belt strength constraint is satisfied. However, to provide some extra capacity, or compensate for some belt wear, or a slight decrease in friction coefficient, we usually select a larger value, perhaps 8, or even 10 in. Selecting $c = 8$ in., for example, we would obtain a 1.09 factor of safety FS , from:

$$FS = \frac{F_{allowable}}{F_{design}} = 35/32 = 1.09 \quad (8.26)$$

Factor of safety is a term used to express ratio of loads or stresses as exemplified in equation (8.26). Factors of safety less than one mean that the design will not support the load without "failing." Factors of safety greater than one indicate that the design is "safe." Factors of safety are also discussed later in the chapter.

Finding feasible values for the design variables is what generating and analyzing is all about. **Infeasible designs** are those whose design variable values do not satisfy the constraints. We found that when $d_2 = 2$ in. and $c = 8$ in., all the constraints are satisfied (speed, torque, and tension). But are these feasible values the best values?

Trade-offs As shown in Table 8.5, as we increase the center distance, we obtain lower values of belt tension, and consequently higher factors of safety. And that's good. But, we are also increasing the size of the system. Unfortunately this design problem, like most design problems, exhibits a **trade-off**, wherein one attribute improves as the other degrades. Trade-offs are caused by the interdependency of variables, typically referred to as **coupling**. Both compactness and belt tension are coupled to the center distance. We trade-off higher belt tensions as we satisfy the customer's desire for compactness. We could ask, "What is the best design?" Is more safety (lower belt tension) and a less compact system better than less safety and more compactness?

This question is difficult to answer. However, as it has been said, "beauty is in the eye of the beholder." Why not consider the trade-offs from the customer's perspective. That is, how does the customer feel about the importance of each of these attributes? Also, is there a way to assess overall satisfaction of the customer? This is the process of evaluating, which we consider in the next section.

8.3.3 Evaluating

During evaluation, we choose the best design candidate from among those few designs that we generated and analyzed earlier. Often our company will select the least expensive design. Other times we may want to maximize an aspect of system performance to obtain the fastest, or the lightest, or the most fuel-efficient design. Like many design teams, we could, therefore, select one of these criteria as the measure of "best," to determine our final recommendation.

Most design problems, however, have multiple attributes or criteria that usually involve trade-off decisions. In the belt-pulley system, for example, we need to make compromises between center distance and belt tension (i.e., compactness versus safety).

Therefore, we need a rational method that can help us make these necessary compromises among multiple criteria. As in concept design and configuration design, the weighted-rating method can be used and is recommended whenever possible.

Step 1: Establish a set of evaluation criteria.

Step 2: Rate the feasible designs for each criterion.

Step 3: Weight the ratings according to importance.

Step 4: Sum the weighted ratings to calculate an overall weighted rating.

Step 1. Establish a set of evaluation criteria, importance weights, and satisfaction levels. If we wish to maintain high standards of quality we need to incorporate the "voice of the customer." Therefore, evaluation criteria are often developed from the list of solution evaluation parameters, or from engineering characteristics. Recall, that we use the house of quality, and/or other marketing research tools, to identify the customer's: (1) functional requirements for the product, (2) key engineering characteristics (that measure how well the functions are performed), and (3) the importance of each functional requirement.

For our belt-pulley system, let's assume that the product development team determined that as long as the motor horsepower was fully utilized, the two most important engineering characteristics were belt tension and center distance (compactness). We identified these earlier during the parametric design formulation step as solution evaluation parameters.

Let's further assume that the team was also able to determine that the customer considers belt tension as "very important" and center distance as "important." Assume that we interpret "very important" with a weight of 0.6 and "important" with a weight of 0.4. Note that we can certainly use other numerical values for importance levels. The very act of choosing numerical values often leads to lots of discussion among design team members. And while it may be time-consuming and frustrating, team discussion is very constructive in that it leads to a better understanding of the customer's desires.

Step 2. Rate the feasible designs for each criterion. We can use the weighted rating method and the rating scale of 0–4 (unsatisfactory to very good), as we

Engineering Design

used in Table 4.12. Note that the people doing the testing may or may not reflect the voice of the customer; we can use a customer satisfaction curve or function rather than using let minimum be equal to the number 1. These values of 0 and maximum satisfaction are equal to the main difference is that Rather than arbitrarily setting satisfaction before, we link the value of each criterion, or SEP function. We arbitrarily set minimum satisfaction versus SEP values similar to the ratings of 0 and 4, discussed the value of satisfaction versus SEP values to a value of 0 and maximum satisfaction and curve-fits. Let's see how we can use these to facilitate our belt-pulley designs.

Let's assume that the product development team also prepared estimates to evaluate our belt-pulley design with respect to belt tension and center distance as shown in Figure 8.8.

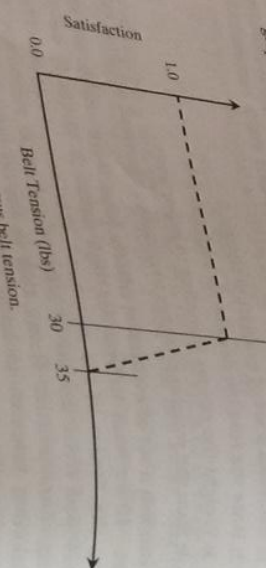


FIGURE 8.7 Customer

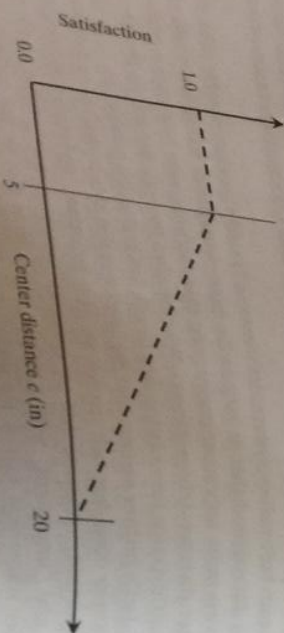


FIGURE 8.8 Customer satisfaction versus center distance.

After the satisfaction curves are determined, we can rate each feasible design by reading values off the curves and entering them into the weighted-rating table.

To facilitate the weighted-rating calculations we can find analytical formulas that fit the curves (i.e., curve-fits) and then automate the computations

spreadsheet. Basic formulas from geometry can be used as in our example. Special computer programs can be used. And Excel's Trendline feature is quite capable in curve-fitting the following relationships: linear, logarithmic, exponential, and power. The relationships are straight lines in the

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \quad (8.2)$$

$$(y - y_1) = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \quad (8.27)$$

We substitute the satisfaction evaluation parameter for S , noting that the maximum value of S is 1, and the minimum value of S is 0. For an SEP with a decreasing satisfaction curve we find the satisfaction as

$$S_x = \frac{(x_2 - x)}{(x_2 - x_1)} \quad (8.25)$$

$$S_{x-} = \frac{(x_2 - x)}{(x_2 - x_1)} \quad (8.28)$$

For an SEP with an increasing satisfaction curve, we find the satisfaction as

$$S_{x+} = \frac{(x - x_1)}{(x_2 - x_1)} \quad (8.29)$$

Note that equations (8.28) and (8.29) are only valid between x_2 and x_{12} .

We set up the bell-pulley saturations in equations (8.30) and (8.31)

$$\xi_c = \frac{(35 - F_1)}{F_{AS}} \quad (8.30)$$

$$S_{F_1} = \frac{(35 - F_1)}{(35 - 30)} \quad (8.30)$$

$$S_c = \frac{(20-c)}{(20-5)} \quad (8.31)$$

$$S_c = \frac{(20 - c)}{(20 - 5)} \quad (8.31)$$

Step 3. Weight the rating according to importance. In this step we multiply satisfaction rating by the respective importance weight.

Step 4. Sum the weighted ratings to calculate an overall weighted rating. We can estimate the customer's overall satisfaction Q as the sum of the importance-weighted satisfaction levels. Specifically, let S_i be the level of the customer's satisfaction for i -th solution evaluation parameter and w_i be equal to the importance weight for that parameter. Then the overall customer satisfaction can be estimated by

$$Q = \sum w_1 S_1 + w_2 S_2 + \dots \quad (8.32)$$

For a center distance equal to 6 in. we can calculate the overall satisfaction as

$$Q = \sum_i w_i S_i = w_F S_F + w_C S_C \quad (8.33)$$

$$Q = 0.6 \frac{(35 - F_1)}{(35 - 30)} + 0.4 \frac{(20 - c)}{(20 - 5)} \quad (8.34)$$

$$Q = 0.6 \frac{(35 - 33.5)}{(35 - 30)} + 0.4 \frac{(20 - 6)}{(20 - 5)} \quad (8.35)$$

$$Q = 0.6(0.3) + 0.4(0.93) = 0.18 + 0.37 = 0.55 \quad (8.36)$$

Other values of overall satisfaction are calculated and summarized in Table 8.6. Note how values of the design variable, c , are coupled to the values of belt tension and compactness. Also, note that equation (8.34) can only be used when both belt tension and center distance are within their respective interpo-

lating points. Outside weight, respective importance values are also plotted in Figure 8.9. The satisfaction curve for belt tension illustrates how as c increases, the customer is more satisfied. The satisfaction illustrates the belt tension decreases, resulting in higher

The overall satisfaction illustrates the belt tension decreases, resulting in higher curve for belt tension because, as c increases, the customer is less satisfied. This occurs because, as c increases, the customer is less satisfied. This occurs because, as c increases, the customer is less satisfied. This occurs because, as c increases, the customer is less satisfied.

customer's satisfaction is somewhat maximized when the center distance is about 10 in. Also, the figure indicates that the overall satisfaction is fairly insensitive to the value of c between 9 and 14 in. The overall satisfaction formula can be used as an aggregate objective function to optimize the belt-pulley design when more than one criterion exists. In other words for **multi-attribute optimization**. **Aggregate objective functions** combine a number of separate objective functions into one scalar function.

TABLE 8.6 Weighted Satisfaction Based On Different Center Distances c

Center distance c (in.)	Tension		Compactness		Overall Satisfaction S
	F_1 (lbs)	S_{F_1}	$w_{F_1} S_{F_1}$	S_c	
4.96	35.00	0.00	0.00	1.00	0.40
6	33.50	0.30	0.18	0.93	0.55
8	32.00	0.60	0.36	0.80	0.68
10	31.25	0.76	0.46	0.67	0.72
14	30.40	0.92	0.55	0.40	0.71
18	30.00	1.00	0.60	0.13	0.65
20	29.90	1.00	0.60	0.00	0.60

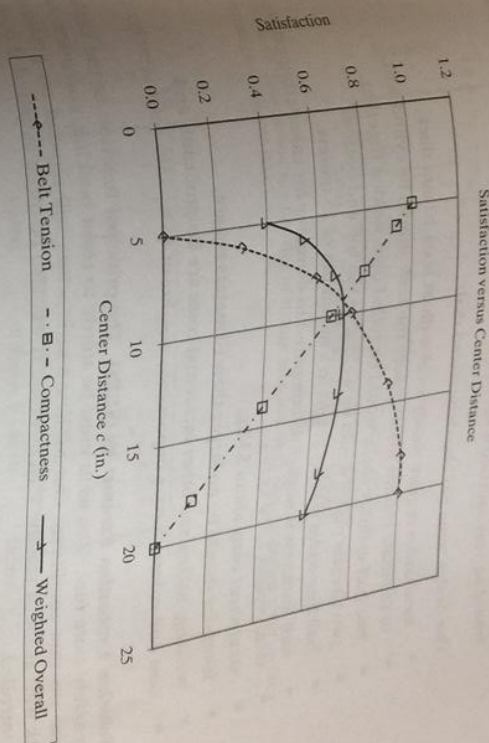


FIGURE 8.9 Satisfaction curves for belt-pulley system.

Using Excel's Solver feature and the overall satisfaction formula as an aggregate objective function, an optimal value of $c = 11.38$ in. resulted in an overall satisfaction of 0.723, whereas $c = 10$ in. results in an overall satisfaction of 0.717. The set of the design variable values that satisfies all the constraints (speed, torque, and tension) and maximizes the customer's satisfaction is therefore, $d_2 = 2$ in. and $c = 11.38$ in.

8.4 SYSTEMATIC PARAMETRIC DESIGN: STORAGE TANK EXAMPLE

Let's consider a classical problem that continues the use of spreadsheets but also involves using the necessary and sufficient conditions for finding the optimum of a function of one variable.

8.4.1 Design Problem Formulation

Our company's Canadian office has asked our engineering team to design an above-ground fuel storage tank. The tank should hold a minimum of 10,000 liters. However, they would be very happy if they could afford a tank holding

seaming Design Their budget is limited to a maximum of \$10,000, but 15,000 liter or more. The tank would cost less than \$5,000, would be very satisfied if the tank should be cylindrical, the tank should be welded together.

The design of the rollers requires that the rollers be rolled to the top and bottom of the container.

- [illegible]

[illegible][illegible]

satisfaction we may trade a higher Q as the sum of the example we may trade a higher Q as the sum of the more storage volume (resulting in more satisfaction) for a customer's overall satisfaction Q as the sum of the cost and volume as

$$Q = \sum W_i S_i = W_C S_C + W_V S_V \quad (8.37)$$

_____ for Storage Tank _____

Parameter	Symbol	Units	Lower Limit	Upper Limit
1 cost	C	\$	\$5,000	\$10,000
2 volume	V	liter	10,000	—

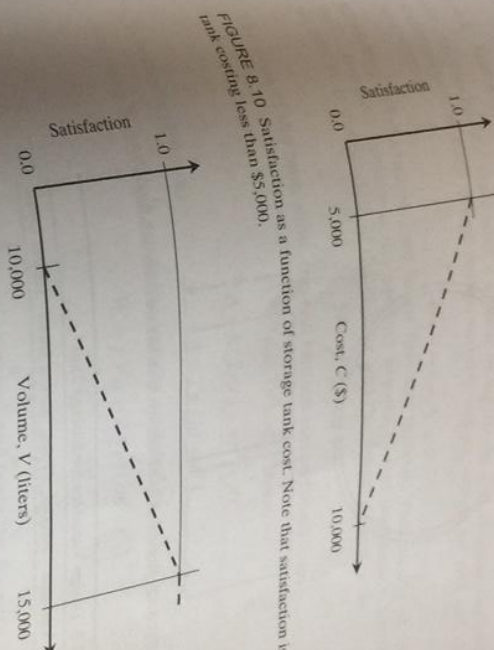


FIGURE 8.10 Satisfaction as a function of storage tank cost. Note that satisfaction is largest for a tank costing less than \$5,000.

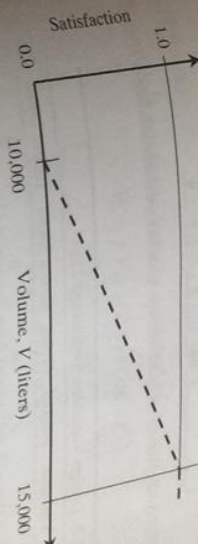


FIGURE 8-11 Satisfaction as a function of storage tank volume. Note that satisfaction is largest for tanks bigger than 15,000 liters.

$$Q = 0.65 \frac{(10,000 - C)}{(10,000 - 5,000)} + 0.35 \frac{(V - 10,000)}{(15,000 - 10,000)} \quad (8.38)$$

Equation (8.38) should be used with caution. The linear interpolation formula should not be used between the end points. This can be handled with an appropriate statement for each term, for example:

$$= \text{IF}(\text{cost} \leq 5000, 65, \text{IF}(\text{cost} > 10000, 0, 65 * (10000 - \text{cost}) / (10000 - 5000))).$$

Design Variables (DV's) The two rolled half-shells are shown in Figure 8.12. The top and bottom circular disks are not shown. The radius r , thickness, t and height h are variables that determine the cost of the tank. Local codes restrict the wall thickness to 10 mm. Therefore the controllable variables are shown in Table 8.12. They are therefore defined as the design variables as shown in Table 8.12.

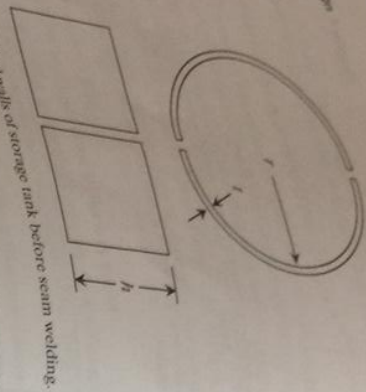


FIGURE 8.12 Rolled walls of storage tank before seam welding.

Design Variable	Symbol	Units	Lower Limit	Upper Limit
1 radius	r	m	0	5
2 height	h	m	0	15

Problem Definition Parameters (PDPs) The givens for this design problem include the unit cost, density of steel and wall thickness (local standard). We identify these as the problem definition parameters in Table 8.9.

Plan for Solving We will use a number of analytical relations from math and physics to estimate values of cost and volume as a function of radius and height. Then we will check that the constraints are not violated. Specifically, we will make sure that the thickness is less than 10 (mm) and minimum volume of 10,000 liters, the footprint is less than 10 (m²) and the \$10,000 maximum cost are not violated.

Parameter	Symbol	Units	Value
1 material cost	c_M	\$/kg	3.00
2 weight density	γ	Kg/m ³	7,830
3 wall thickness	t	mm	10

Generating, Analyzing and Evaluating

We generate a model of the tank with a system of equations, and thereby analyze the model. We evaluate the model by checking the constraints, which we will call design constraints.

Modeling For example, we know that the material volume V_M of the tank walls, top and bottom is the product of area A and thickness t . Further, the weight W is the product of the volume and the weight density γ as

$$W = \gamma V_M = \gamma t A \quad (8.39)$$

The area of the tank walls, top and bottom can be found from

$$A = A_{\text{walls}} + A_{\text{top}} + A_{\text{bottom}} \quad (8.40)$$

$$A = h(2\pi r) + \pi r^2 + \pi r^2 \quad (8.41)$$

$$A = 2\pi (hr + r^2) \quad (8.42)$$

Which simplifies to

$$W = \gamma V_M = 2\pi \gamma t (hr + r^2) \quad (8.43)$$

$$C = c_M W \quad (8.44)$$

$$C = c_M 2\pi \gamma t (hr + r^2) \quad (8.45)$$

The storage volume, V can be calculated from the formula for the volume of a right circular cylinder as

$$V = h\pi r^2 \quad (8.46)$$

From equation (8.45) and (8.46) we see that cost and volume are directly related to the radius and the height. We note that this design problem involves two equations and two unknowns. For a tank of constant volume, we can eliminate height, h by substituting equation (8.46) into (8.45) obtaining

$$C = c_M 2\pi \gamma t \left(\left(\frac{V}{\pi r^2} \right) r + r^2 \right) \quad (8.47)$$

$$C = c_M 2\pi \gamma t \left(\frac{V}{\pi r} + r^2 \right) \quad (8.48)$$

Which simplifies to

Since all the parameters in equation (8.48) are constants except for radius, it is a function of one variable. Using the necessary condition from calculus for determining a stationary point we can take the first derivative of the cost function with respect to the radius and set it to zero to obtain

$$\frac{dC}{dr} = \frac{d}{dr} \left(c_M 2\pi \gamma t \left(\frac{V}{\pi r} + r^2 \right) \right) = 0 \quad (8.49)$$

$$\frac{dC}{dr} = c_w 2\pi r t \left(-\frac{V}{\pi r^3} + 2r \right) = 0 \quad (8.50)$$

$$r^* = \left(\frac{V}{2\pi} \right)^{1/3} \quad (8.51)$$

Finding the optimal radius as an optimum to test the second

We next use the sufficient condition for an optimum to test the second derivative as

$$\frac{d^2C}{dr^2} = \left(\frac{2V}{\pi r^3} + 2 \right) > 0 \quad (8.52)$$

Since the second derivative of the cost function is greater than zero for positive, non-zero values of r , we readily conclude that the stationary point r^* is a minimum. This behavior is illustrated in Figure 8.13.

Analyzing and Evaluating Recognizing that a minimum weight tank can be designed using the optimal radius and height, we can develop a spreadsheet to compute the weight, cost and weighted satisfactions as a function of volume. As shown in Table 8.10, we see that as the volume of the tank increases, the weight and cost increase. The weighted satisfaction for cost decreases, as the tank becomes more expensive. If we were to merely minimize cost, we would select a 10,000 liter design with radius 1.17 (m) and height 2.34 (m). This decision would result in an overall satisfaction of 0.52. The weighted satisfaction for volume increases with larger tank size. Which tank size is the "best?"

15,000 (liter) or more. Note how these trade-off. Which tank size is the "best?"

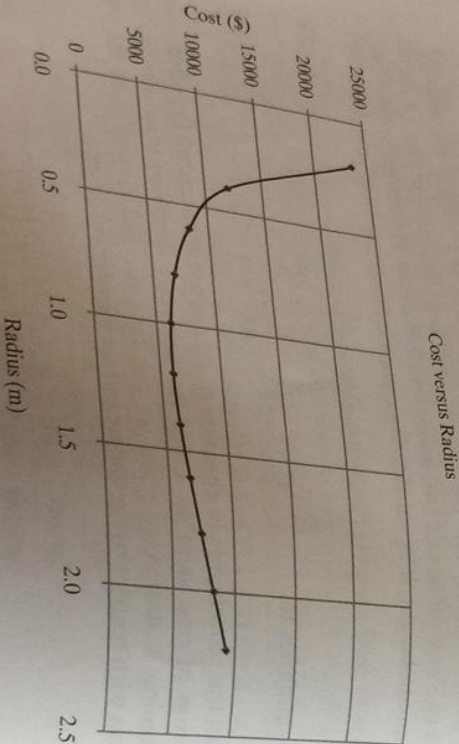


FIGURE 8.13 The cost for a 10,000 liter tank reaches a minimum for a radius of 1.17 (m). The optimal radius r^* can be calculated using equation (8.51).

TABLE 8.10 Storage Tank Design

TABLE 8.10 Design Parameters				
Problem Definition Parameters		Solution Evaluation Parameters		
Parameter	Symbol	Units	Value	
cost/unit wt.	c_w	\$/kg	3	
cost/density	V	kg/m ³	7830.0	
wt thickness	t	mm	10	
Design Variables		Symbol	Units	Lower Upper
Variable	r	m	0	5
radius	h	m	5	15
height				
Solution Evaluation Parameters		Symbol	Units	Lower Upper
Parameter	C	\$	5,000	10,000
cost	V	m ³	10,000	none
volume				
Performance calculations				
$V(l)$	$r^*(m)$	$h^*(m)$	$W(kg)$	$C(\$)$
10000	1.17	2.34	2011.92	6035.75
11000	1.21	2.41	2143.90	6431.70
11000	1.24	2.48	2271.94	6815.82
12000	1.27	2.55	2396.47	7189.41
13000	1.31	2.61	2517.84	7553.52
14000	1.34	2.67	2636.35	7909.06
15000	1.37	2.73	2752.26	8256.78
16000	1.39	2.79	2865.78	8597.33
17000	1.42	2.84	2977.09	8931.26
18000	1.45	2.89	3086.35	9259.05
19000				
20000	1.47	2.94	3193.72	9581.15
Satisfaction				
	w_{Sc}	w_{Sv}	Q	
	0.52	0.00	0.52	
	0.46	0.07	0.53	
	0.41	0.14	0.55	
	0.37	0.21	0.58	
	0.32	0.28	0.60	
	0.27	0.35	0.62	
	0.23	0.35	0.58	
	0.18	0.35	0.53	
	0.14	0.35	0.49	
	0.10	0.35	0.45	
	0.05	0.35	0.40	

If we were to make our final decision solely based on a single attribute, we would select a 10,000 (liter) tank size. To maximize total satisfaction we should use overall satisfaction Q , which is a sum of the two weighted satisfaction values. Overall satisfaction reaches a maximum of 0.62 at a tank size of 15,000 (liter). This reflects the increase in volume for a marginal increase in cost. A tank larger than 15,000 (liter), however, does not improve satisfaction because the customer is indifferent (no change in satisfaction) for tanks larger than 15,000 (liter). Therefore we select a 15,000 (liter) tank with radius 1.34 (m), height 2.67 (m) and a cost of \$7,090. These trends are illustrated in Figure 8.14.

Lastly, we must also check that constraints are not violated. The thickness is equal to 10 (mm). The cost is less than \$10,000. The tank volume is greater than 10,000 (liter). And the footprint is $\pi r^2 = \pi(1.34)^2 = 5.6$ (m²). Our selected design maximizes satisfaction and does not violate constraints.

