

MATH 1342 – STATISTICS

SECTION 7.1: THE CENTRAL LIMIT THEOREM FOR SAMPLE MEANS (AVERAGES)

- The **central limit theorem (CLT)** for sample means says that if you keep drawing larger and larger samples and calculating their means, the sample means form their own normal distribution (the sampling distribution).

- Thus, in a population whose distribution X may be known or unknown, if the size n of samples is sufficiently large, the distribution of the sample means $\bar{X}$ will be approximately normal.

- $\bar{X} \sim N\left(\mu_x, \frac{\sigma_x}{\sqrt{n}}\right)$ $\bar{X} \sim N(\mu_{\bar{X}}, \frac{\sigma_{\bar{X}}}{\sqrt{n}})$ $\mu_{\bar{X}} = \mu_x$ (the mean of the sample means will equal the population mean)

Greek letters $\Rightarrow$ population

- Standard error of the mean:** $\sigma_{\bar{X}} = \frac{\sigma_x}{\sqrt{n}}$ = standard deviation of $\bar{X}$

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

- The variable n is the number of values that are averaged together, not the number of times the experiment is done.

- The random variable $\bar{X}$ has a different z -score associated with it from that of the random variable X . The mean $\bar{X}$ is the value of $\bar{X}$ in one sample: $z = \frac{\bar{x} - \mu_{\bar{x}}}{\left(\frac{\sigma_{\bar{x}}}{\sqrt{n}}\right)}$ where $\mu_{\bar{x}}$ is the average of both X and $\bar{X}$.

For means, $\bar{X} \sim N\left(\mu_x, \frac{\sigma_x}{\sqrt{n}}\right)$
To Find a Probability (Page 397)

(1.) Go to [2nd] [DISTR]

(2.) 2:normalcdf

(3.) normalcdf (lower value, upper value, mean, $\frac{\text{std dev}}{\sqrt{\text{sample size}}}$)

$$\bar{X} \sim N(\mu_{\bar{x}}, \sigma_{\bar{x}})$$

To Find a Percentile (Page 393)

(1.) Go to [2nd] [DISTR]

(2.) 3:invNorm

(3.) invNorm (area to left, mean, $\frac{\text{std dev}}{\sqrt{\text{sample size}}}$)

Ex. 1: Cans of a cola beverage claim to contain 16 ounces. The amounts in a sample are measured and the statistics are $n = 34$, $\bar{X} = 16.01$ ounces. If the cans are filled so that $\mu = 16.00$ ounces (as labeled) and $\sigma = 0.143$ ounces, find the probability that a sample of 34 cans will have an average amount greater than 16.01 ounces. Do the results suggest that cans are filled with an amount greater than 16 ounces?

$$\begin{aligned} \mu &= 16.00 = \mu_{\bar{x}} \\ \sigma &= .143 \Rightarrow \sigma_{\bar{x}} = \frac{.143}{\sqrt{34}} \\ n &= 34 \end{aligned}$$

$$\begin{aligned} \bar{X} &\sim N(\mu_{\bar{x}}, \sigma_{\bar{x}}) \\ \bar{X} &\sim N(16.00, \frac{.143}{\sqrt{34}}) \end{aligned}$$

$$P(\bar{X} > 16.01) = \text{normalcdf}(16.01, 1E99, 16, \frac{.143}{\sqrt{34}})$$

$$\approx .3417$$

There is a 34.17% chance that a sample of 34 cans has average amount greater than 16.01 oz. Customers Manufacturers



$\mu = 8.2$

$\sigma = 1$

$n = 60$

Ex. 2: The mean number of minutes for app engagement by a tablet user is 8.2 minutes. Suppose the standard deviation is one minute. Take a sample of 60.

(a) What are the mean and standard deviation for the sample mean number of app engagement by a tablet user?

$$\mu_{\bar{x}} = \mu = 8.2$$

$$\sigma_{\bar{x}} = \frac{1}{\sqrt{60}}$$

(b) What is the standard error of the mean? = std deviation for $\bar{x}$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{60}}$$

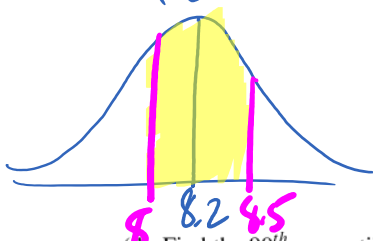
Thus allows us to calculate the probability of sample means of a population distance from the mean relative to a sample size of 60.

(c) Find the probability that the sample mean is between eight minutes and 8.5 minutes.

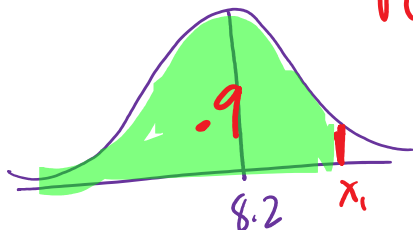
$$P(8 < \bar{x} < 8.5) = \text{normalcdf}(8, 8.5, 8.2, \frac{1}{\sqrt{60}})$$

$$\approx .9293 \leftarrow \text{prob.}$$

Enter as a percent: 92.93%



(d) Find the 90th percentile for the sample mean time for app engagement for a tablet user. Interpret this value in a complete sentence.



$$P(x < x_1) = .9$$

$$x_1 = \text{invnorm}(.9, 8.2, \frac{1}{\sqrt{60}})$$

$$x_1 \approx 8.37$$

90% of average app engagement time for a sample of 60 tablet users is 8.37 minutes or less