

Question #8:

$$A = \begin{bmatrix} 2x+1 & x & 3x+2 \\ 0 & 4 & 0 \\ 4x-4 & 1 & 6x-5 \end{bmatrix}$$

$$AI = \left[\begin{array}{ccc|ccc} 2x+1 & x & 3x+2 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 4x-4 & 1 & 6x-5 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{x}{2x+1} & \frac{3x+2}{2x+1} & \frac{1}{2x+1} & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 4x-4 & 1 & 6x+5 & 0 & 0 & 1 \end{array} \right]$$

Divide row 1 by
 $2x+1$ ($R_1 = \frac{R_1}{2x+1}$)

$$\left[\begin{array}{ccc|ccc} 1 & \frac{x}{2x+1} & \frac{3x+2}{2x+1} & \frac{1}{2x+1} & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2x+1}(-4x(x-1)+2x+1) & \frac{3}{2x+1} & \frac{4(1-x)}{2x+1} & 0 & 1 \end{array} \right] \begin{array}{l} \text{Subtract row 1} \\ \text{multiplied by } 4x-4 \\ \text{from row 3} \\ (R_3 = R_3 - (4x-4)R_1) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{x}{2x+1} & \frac{3x+2}{2x+1} & \frac{1}{2x+1} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{2x+1}(-4x(x-1)+2x+1) & \frac{3}{2x+1} & \frac{4(1-x)}{2x+1} & 0 & 1 \end{array} \right] \begin{array}{l} \text{Divide row 2 by 4} \\ (R_2 = \frac{R_2}{4}) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3x+2}{2x+1} & \frac{1}{2x+1} & -\frac{x}{8x+4} & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{2x+1}(-4x(x-1)+2x+1) & \frac{3}{2x+1} & \frac{4(1-x)}{2x+1} & 0 & 1 \end{array} \right] \begin{array}{l} \text{Subtract row 2} \\ \text{multiplied by } \frac{x}{2x+1} \\ \text{from row 1} \\ (R_1 = R_1 - \left(\frac{x}{2x+1}\right)R_2) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3x+2}{2x+1} & \frac{1}{2x+1} & -\frac{x}{8x+4} & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{3}{2x+1} & \frac{4(1-x)}{2x+1} & \frac{4x^2-6x-1}{4(2x+1)} & 1 \end{array} \right] \begin{array}{l} \text{Subtract row 2 multiplied by} \\ \frac{1}{2x+1}(-4x(x-1)+2x+1) \text{ from} \\ \text{row 3} \\ (R_3 = R_3 - (V)R_2) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3x+2}{2x+1} & \frac{1}{2x+1} & -\frac{x}{8x+4} & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{4}{3} - \frac{4x}{3} & \frac{x^2}{3} - \frac{x}{2} - \frac{1}{12} & \frac{2x}{3} + \frac{1}{3} \end{array} \right] \begin{array}{l} \text{multiply row 3 by} \\ \frac{2x}{3} + \frac{1}{3} \\ (R_3 = \left(\frac{2x}{3} + \frac{1}{3}\right)R_3) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2x - \frac{5}{3} & -\frac{x^2}{2} + \frac{2x}{3} + \frac{1}{6} & -x - \frac{2}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{4}{3} - \frac{4x}{3} & \frac{x^2}{3} - \frac{x}{2} - \frac{1}{12} & \frac{2x}{3} + \frac{1}{3} \end{array} \right] \begin{array}{l} \text{Subtract row 3} \\ \text{multiplied by } \frac{3x+2}{2x+1} \\ \text{from row 1} \\ (R_1 = R_1 - \left(\frac{3x+2}{2x+1}\right)R_3) \end{array}$$

Therefore,

$$A^{-1} = \begin{bmatrix} 2x - \frac{5}{3} & -\frac{x^2}{2} + \frac{2x}{3} + \frac{1}{6} & -x - \frac{2}{3} \\ 0 & \frac{1}{4} & 0 \\ \frac{4}{3} - \frac{4x}{3} & \frac{x^2}{3} - \frac{x}{2} - \frac{1}{12} & \frac{2x}{3} + \frac{1}{3} \end{bmatrix}$$