Math 1149 - Second Midterm exam

First and last name (please, print): _____

OSU email address (please, print): _____

Here are the rules for this exam.

1. You have **24 hours** to complete this exam.

2. **If you are writing this exam on your tablet, fill-in your full name and your OSU email address** in the space provided above, and write the solutions to each exercise in the space provided after each exercise. **If you are writing this exam on paper, put your name and OSU email address at the beginning of your papers.**

3. **There are 6 questions in total**, most having multiple parts. For full credits, **show all your work**, that is, make sure that what you write is enough to understand your solutions and thinking process. Unless otherwise stated, **you need to give exact answers**, and all **answers should be simplified**.

4. This is an **open-book exam** (you can consult the class material and the textbook), on which you are **supposed to work on your own, without online aids and without aids from other people. By taking and turning in this exam, you pledge your honor to abide by these rules. Breaking these rules is considered academic dishonesty, and will be treated accordingly.**

5. **Write legibly**.

Do not panic and good luck!

There are **six questions** in total. For full credits, and unless otherwise stated, **show all your work, and give exact and simplified answers only.**

1. (16 points) Consider the function $f(x) = 2\sin\left(\frac{\pi x}{4} - \frac{\pi}{2}\right)$.

   (a) (1 point) Find the amplitude of $f(x)$.

   (b) (2 points) Find the range and the period of $f(x)$.

   (c) (3 points) Find a full-cycle interval of $f(x)$.

   (d) (6 points) Find the x-values of the key-points of $f(x)$ on the full-cycle interval found in part (c) above. Then, compute the values of $f(x)$ at the first two of those key-points.

   (e) (4 points) Use the graph paper below to sketch the graph of $f(x)$ on a full-cycle interval. Mark the key-points clearly.
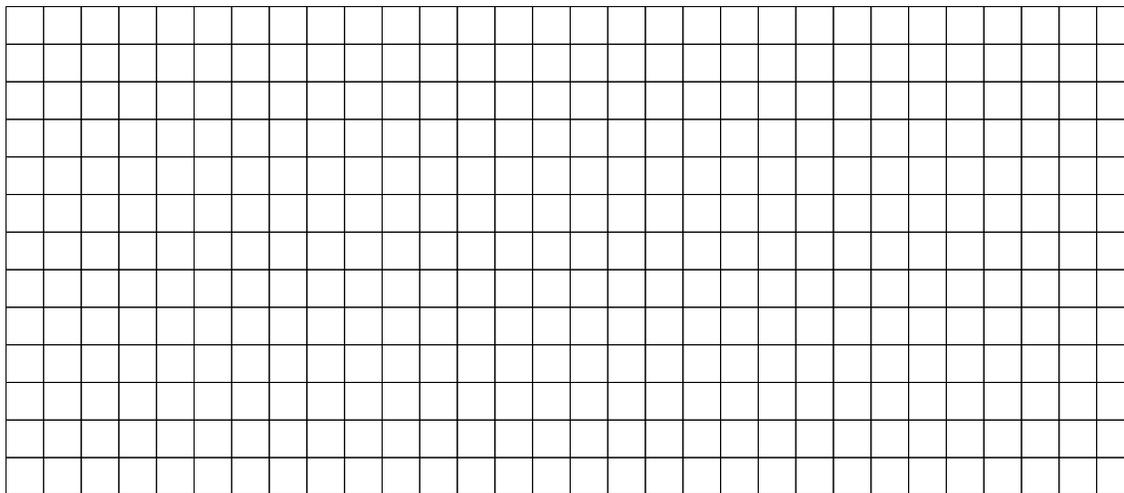
2. (18 points) Find the exact values of each of the following expressions. Make sure to explain how you found your answers.

(a) (3 points) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

(b) (4 points) $\arctan\left(\tan\left(\frac{11\pi}{6}\right)\right)$

(c) (4 points) $\cos^{-1}\left(\cos\left(\frac{19\pi}{18}\right)\right)$

(d) (7 points) $\sin\left(\tan^{-1}\left(-\frac{\sqrt{19}}{3}\right)\right)$

3. (12 points) Rewrite the following expression as an algebraic expression:

$$\tan(\arcsin(3x-1)), \quad \text{for} \quad \frac{1}{3} < x < 1.$$

You do *not* need to rationalize your final answer.

4. (17 points) Simplify each of the following expressions to a form that has no fractions. Make sure to show all the steps of your computations.

   (a) (4 points)
   $$\frac{\cos^3 x}{\sin(x) + 1}.$$

   (b) (6 points)
   $$\frac{1 + \tan(x)}{\sin(x) + \cos(x)}.$$

   (c) (7 points)
   $$\frac{\csc^4(x) - \cot^4(x)}{2\cot^2(x) + 1}$$

   (Hint: rewrite the numerator by using polynomial factorizations and trigonometric identities.)

5. (12 points) Verify that the following equations are identities

(a) (5 points)

$$\cot(x)\sec^2(x) - \cot(x) = \tan(x).$$

(b) (7 points)

$$\frac{\sin(x)}{\cot(x) - \csc(x)} = -\cos(x) - 1.$$

6. (15 points) Suppose given angles α and β such that:

- α is on the second quadrant and $\sin(\alpha) = \frac{\sqrt{3}}{3}$;
- β is on the third quadrant and $\cos(\beta) = -\frac{5}{6}$.

(a) (5 points) Find $\cos(\alpha)$.

(b) (5 points) Find $\sin(\beta)$.

(c) (5 points) Use the appropriate sum or difference formula to find $\cos(\alpha - \beta)$.

Blank Page

Use this page if you need more space for your computations.
If you want something you write here to get marked, make it clear which exercise your work refers to.