

$$\begin{aligned}
 1. \quad f(x) &= -x^2 - 4x + 5 \\
 &= -(x^2 + 4x - 5) \\
 &= -(x^2 + 2x + 4 - 5 - 4) \\
 &= -(x+2)^2 + 9
 \end{aligned}$$

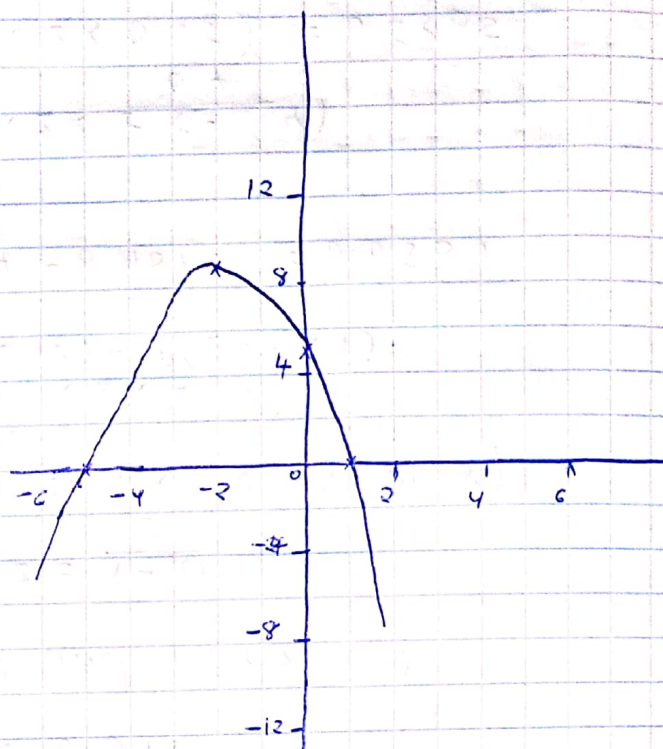
$$\text{Vertex} = (h, k) = (-2, 9)$$

$$\begin{aligned}
 \text{y-intercept} &= -(0)^2 - 4(0) + 5 \\
 &= 5 \\
 &= (0, 5)
 \end{aligned}$$

$$\text{Pt sym to y-intercept} = (-4, 5)$$

x-intercepts

$$\begin{aligned}
 -x^2 - 4x + 5 &= 0 \\
 \cancel{-x^2} - \cancel{x} - 5x + 5 &= 0 \\
 \cancel{-x(x+1)} - 5 &= 0 \\
 -(x^2 + 4x - 5) &= 0 \\
 -(x-1)(x+5) &= 0 \\
 x=1, x=-5 \\
 (1, 0), (-5, 0)
 \end{aligned}$$



$$2. \quad f(x) = x^3 + 2x^2 - 4x - 8$$

End behaviour = Falls on far left
Rises on far right

x-intercepts

$$\begin{aligned}
 x^3 + 2x^2 - 4x - 8 &= 0 \\
 x^2(x+2) - 4(x+2) &= 0 \\
 (x+2)(x^2-4) &= 0 \\
 x+2=0 & \quad x^2-4=0 \\
 x=-2 & \quad x^2=4 \\
 & \quad x=-2, 2
 \end{aligned}$$

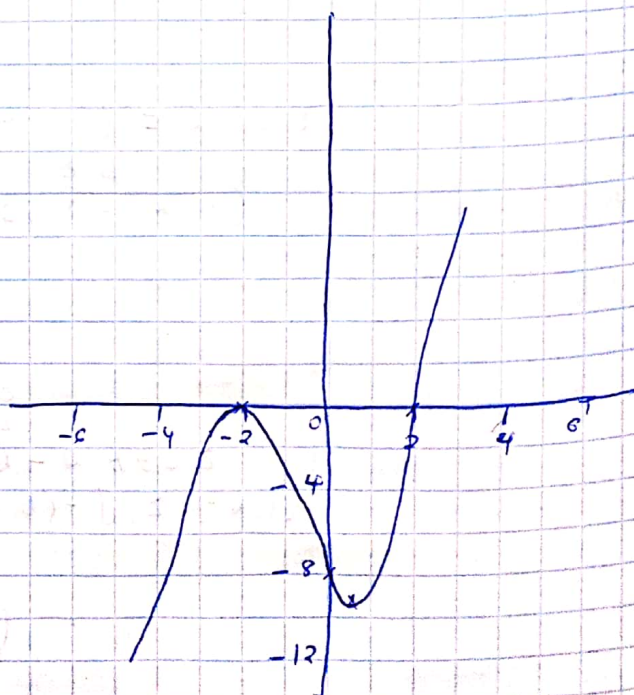
$$= (-2, 0), (-4, 0), (2, 0)$$

$(-2, 0)$ = Turn around
 $(2, 0)$ = Crosses axis

Symmetry = None

$$\text{y-intercept} = (0, -8)$$

$$\text{Max. turning points} = 3 - 1 = 2$$



$$3. f(x) = 3x^2 - 6x + 4$$

Vertex = (h, k)

$$x = -\frac{b}{2a} = \frac{6}{2(3)} = 1$$

$$f(1) = k = 3(1)^2 - 6(1) + 4 = 1$$

Vertex = (1, 1)

Since $a > 0$, the vertex is a minimum value.

Minimum value = 1
 $x = 1$

$$4. f(x) = (x+4)(x+2i)(x-2i) \\ = (x+4)(x^2 - 2i + 2i + 4) \\ = (x+4)(x^2 + 4) \\ = x^3 + 4x + 4x^2 + 16$$

$$f(x) = x^3 + 4x^2 + 4x + 16$$

5.

$$\begin{array}{r|rrrr} 3 & 2 & -5 & -1 & -6 \\ & \downarrow & 6 & 3 & 6 \\ \hline & 2 & 1 & 2 & 0 \end{array}$$

$f(3) = 0$ Is $(x-3)$ a factor? YES

$$6. f(x) = \frac{2x^2}{x^2 - 4}$$

Vertical asymptote

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = -2, x = 2$$

Horizontal asymptote
 $y = 2$

y-intercept

$$y = \frac{2(0)^2}{0^2 - 4} = 0 \\ = (0, 0)$$

x-intercept

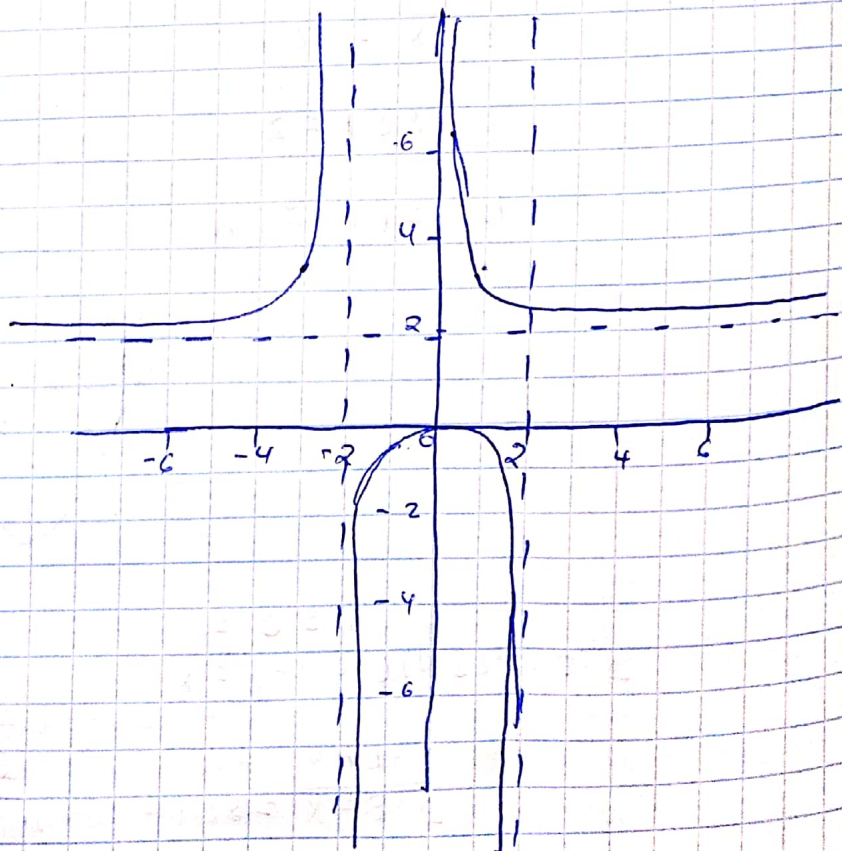
$$\frac{2x^2}{x^2 - 4} = 0$$

$$x^2 - 4$$

$$2x^2 = 0$$

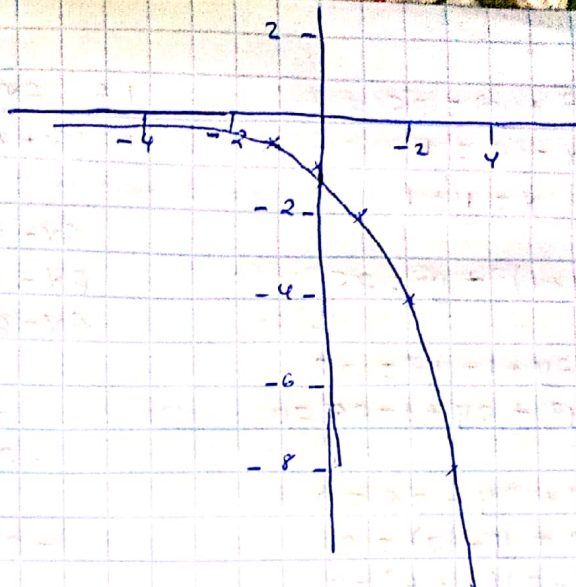
$$x = 0$$

$(0, 0)$ Other points = $(-3, 3.6), (3, 3.6)$



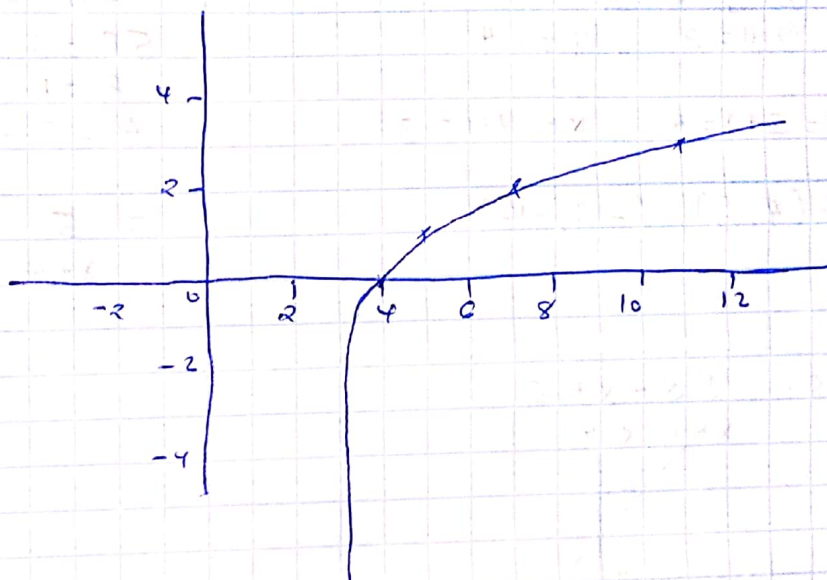
7. $y = -2^x$

x	y
-1	-0.5
0	1
1	-2
2	-4
3	-8



8. $y = \log_2(x-3)$

x	y
4	0
5	1
7	2
11	3



9. $P_0 = P_0 e^{-0.05t}$

$0.5 = e^{-0.05t}$

$\ln(0.5) = -0.05t$

$t = \frac{\ln(0.5)}{-0.05} = 13.86 \text{ yrs}$

10. $\log_5(x-2) + \log_5(x+2) = 1$
 $\log_5(x-2) + \log_5(x+2) = \log_5 5$
 $\log_5((x-2)(x+2)) = \log_5 5$
 $(x-2)(x+2) = 5$
 $x^2 - 4 = 5$
 $x^2 = 9$

$x = -3$ False

$x = 3$ True

Ans: $x = 3$

11. $\ln(x+2) - \ln(x-2) = \ln(3)$

$\ln\left(\frac{x+2}{x-2}\right) = \ln(3)$

$\frac{x+2}{x-2} = 3$

$x-2$

$x+2 = 3x-6$

$3x - x = 2 + 6$

$2x = 8$

$x = 4$

12. $(x - 3y = -30) \quad (-5)$

$-5x - 4y = -2$

$-5x + 15y = 150$

$-5x - 4y = -2$

$19y = 152$

$y = 8$

$x - 3(8) = -30$

$x = -6$

$= (-6, 8)$

$$13. \begin{aligned} x^2 + y^2 &= 25 \\ x - y &= 1 \\ x &= y + 1 \end{aligned}$$

$$(y+1)^2 + y^2 = 25$$

$$\begin{aligned} y^2 + 2y + 1 + y^2 &= 25 \\ 2y^2 + 2y + 1 - 25 &= 0 \end{aligned}$$

$$\begin{aligned} 2y^2 + 2y - 24 &= 0 \\ 2(y^2 + y - 12) &= 0 \\ 2(y-3)(y+4) &= 0 \\ y = 3, & \quad y = -4 \end{aligned}$$

$$x = 3+1 = 4 \quad x = -4+1 = -3$$

$$= (4, 3), (-3, -4)$$

$$14. \frac{6x-6}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2}$$

$$\begin{aligned} 6x-6 &= A(x-2) + B(x+4) \\ 6x-6 &= Ax-2A+Bx+4B \\ 6x-6 &= x(A+B) - 2A+4B \end{aligned}$$

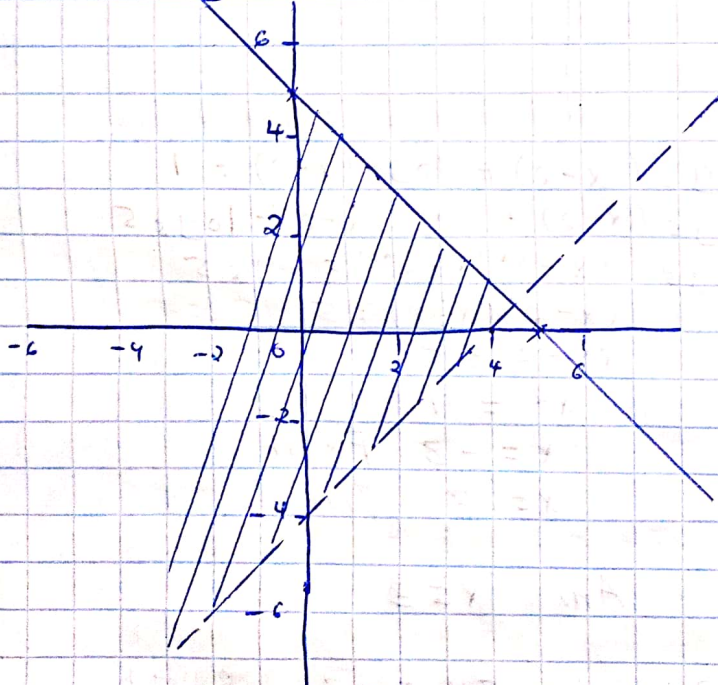
$$\begin{aligned} A+B &= 6 & A &= 6-B \\ -2A+4B &= -6 \end{aligned}$$

$$\begin{aligned} -2(6-B) + 4B &= -6 \\ -12 + 2B + 4B &= -6 \\ 6B &= 6 \\ B &= 1 \end{aligned}$$

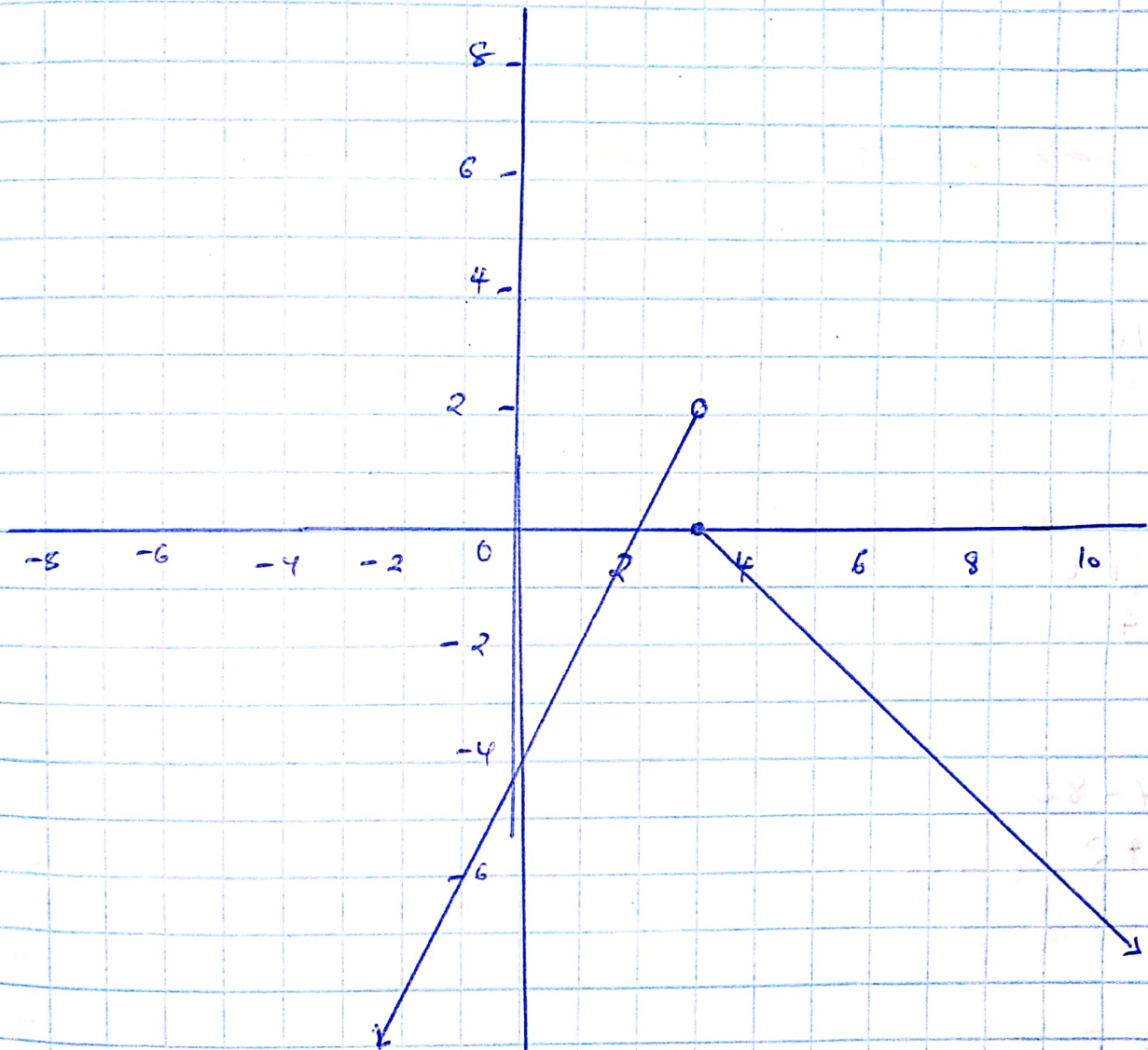
$$A = 6 - 1 = 5$$

$$= \frac{5}{x+4} + \frac{1}{x-2}$$

$$15. \begin{aligned} y &\leq -x + 5 \\ x &\geq y & < 4 \end{aligned}$$



16.



$$17. \left[\begin{array}{cc|c} 3 & -2 & 15 \\ 5 & 10 & 10 \end{array} \right]$$

$$R_2 = R_2 - \frac{5}{3} R_1$$

$$\left[\begin{array}{cc|c} 3 & -2 & 15 \\ 0 & \frac{40}{3} & -35 \end{array} \right]$$

$$3x - 2y = 15$$

$$\frac{40}{3}y = -35$$

$$y = -35 \times \frac{3}{40} = -\frac{21}{8}$$

$$3x - 2\left(-\frac{21}{8}\right) = 15$$

$$3x + \frac{21}{4} = 15$$

$$3x = \frac{39}{4}$$

$$x = \frac{13}{4}$$

$$= \left(\frac{13}{4}, -\frac{21}{8} \right)$$

consistent
independent

$$18. \quad \begin{aligned} 2x + 5y &= 43 \\ 2x + 2y &= 22 \end{aligned}$$

$$D = \begin{vmatrix} 2 & 5 \\ 2 & 2 \end{vmatrix} = 4 - 10 = -6$$

$$D_x = \begin{vmatrix} 43 & 5 \\ 22 & 2 \end{vmatrix} = 86 - 110 = -24$$

$$D_y = \begin{vmatrix} 2 & 43 \\ 2 & 22 \end{vmatrix} = 44 - 86 = -42$$

$$x = \frac{D_x}{D} = \frac{-24}{-6} = 4$$

$$y = \frac{D_y}{D} = \frac{-42}{-6} = 7$$

$$= (4, 7)$$

Consistent Independent

$$19. \quad 2A = 2 \begin{bmatrix} -3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -6 & 10 \\ 2 & 4 \end{bmatrix}$$

$$3B = 3 \begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -6 & 9 \\ 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 10 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -6 & 9 \\ 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -10 & 7 \end{bmatrix}$$

20. $AX = B$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} -2 & 4 & -1 \\ 3 & -5 & 2 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 6 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} -14 + 24 + 6 \\ 21 - 30 - 12 \\ -7 + 12 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 16 \\ -21 \\ 11 \end{bmatrix}$$

$$(X, Y, Z) = (16, -21, 11)$$

21. $A = \begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix}$

$$\begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3a - 4c & 3b - 4d \\ a - 2c & b - 2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3a - 4c = 1$$

$$a - 2c = 0$$

$$a = 2c$$

$$6c - 4c = 1$$

$$2c = 1$$

$$c = \frac{1}{2}$$

$$a = 1$$

$$3b - 4d = 0$$

$$b - 2d = 1$$

$$b = 1 + 2d$$

$$3 + 6d - 4d = 0$$

$$d = -\frac{3}{2}$$

$$b = -2$$

$$= \begin{bmatrix} 1 & -2 \\ \frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

$$\begin{aligned}
 22 \quad & \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = 1(-1-3) - 1(1-6) + 1(1+2) \\
 & = -4 + 5 + 3 = 4
 \end{aligned}$$

23

$$\ln A = \lim_{n \rightarrow \infty} \ln \left[P \left(1 + \frac{r}{n} \right)^{nt} \right] = \lim_{n \rightarrow \infty} \left[\ln P + \ln \left(1 + \frac{r}{n} \right)^{nt} \right]$$

$$= \ln P + \lim_{n \rightarrow \infty} \left[nt \ln \left(1 + \frac{r}{n} \right) \right]$$

$$\ln A = \ln P + \frac{rt}{1+0}$$

$$\ln A - \ln P = rt$$

$$\ln \left(\frac{A}{P} \right) = rt$$

$$\frac{A}{P} = e^{rt}$$

$$A = P e^{rt}$$

24.

$$f(x) = \frac{x}{x+2}$$

$$x = \frac{y}{y+2}$$

$$xy + 2x = y$$

$$xy - y = -2x$$

$$y(x-1) = -2x$$

$$y = \frac{-2x}{x-1}$$

$$f^{-1}(x) = \frac{-2x}{x-1}$$