

QUESTIONS d8e

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$y_t = \mu + u_t$$

d) $\alpha_0 \geq 0$

$$\alpha_i \geq 0$$

$$\beta_j \geq 0$$

✓ To ensure that the unconditional variance is finite

$$\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) \leq 1 \quad \text{i.e.}$$

Max. (p, q)

$$\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1, \quad \alpha_i = 0 \text{ for } i > p$$
$$\beta_j = 0 \text{ for } j > q$$

e

$$\hat{u} = 0.0023$$

$$\hat{\alpha}_1 = 0.1251$$

$$\hat{\alpha}_0 = 0.0172$$

$$\hat{\beta}_1 = 0.8611$$

Time = T

SET OF EQUATIONS

① ONE-STEP AHEAD FORECAST

$$\hat{S}_t^1 = \alpha_0 + \sum_{i=1}^u \alpha_i u_{t+1-i}^2 + \sum_{j=1}^V \beta_j S_{t+1-j}^2 \quad \checkmark$$

② TWO-STEP AHEAD FORECAST

$$\hat{S}_t^2 = \alpha_0 + \sum_{i=1}^u \alpha_i u_{t+2-i}^2 + \sum_{j=1}^V \beta_j S_{t+2-j}^2 \quad \checkmark$$

③ THREE-STEP-AHEAD FORECAST

$$\hat{S}_t^3 = \alpha_0 + \sum_{i=1}^u \alpha_i u_{t+3-i}^2 + \sum_{j=1}^V \beta_j S_{t+3-j}^2 \quad \checkmark$$

DERIVATION OF EQUATIONS

① ONE-STEP-AHEAD-FORECAST

$$\hat{S}_T^1 = \alpha_0 + \alpha_1 u_{t+1-1}^2 + \beta_1 S_{t+1-1}^2$$

$$= \alpha_0 + \alpha_1 u_t^2 + \beta_1 S_t^2$$

$$\checkmark = 0.0172 + 0.1251 u_t^2 + 0.8611 S_t^2$$

$$\hat{S}_T^2 = 0.0172 + 0.1251 U_t^2 + 0.8611 S_T^2$$

And mean Equation becomes

$$Y_t = 0.0023 + U_t$$

TWO-STEP-AHEAD - FORECAST

$$\hat{S}_T^2 = \alpha_0 + \alpha_1 U_{t+2-1}^2 + \beta_1 S_{t+2-1}^2$$

This from the equation below

$$\hat{S}_T^2 = \alpha_0 + \sum_{i=1}^1 \alpha_i U_{t+2-i}^2 + \sum_{j=1}^1 \beta_j S_{t+2-j}^2$$

$$= \alpha_0 + \alpha_1 U_{t+1}^2 + \beta_1 S_{t+1}^2$$

$$\hat{S}_T^2 = 0.0172 + \alpha_1 U_{t+1}^2 + 0.8611 S_{t+1}^2$$

$$\checkmark = 0.0172 + 0.1251 U_{t+1}^2 + 0.8611 S_{t+1}^2$$

THREE-STEP-AHEAD - FORECAST

$$\hat{S}_T^2 = \alpha_0 + \sum_{i=1}^3 \alpha_i U_{t+3-i}^2 + \sum_{j=1}^1 \beta_j S_{t+3-j}^2$$

$$= \alpha_0 + \sum_{i=1}^3 \alpha_i U_{t+3-i}^2 + \sum_{j=1}^1 \beta_j S_{t+3-j}^2$$

$$= \alpha_0 + \alpha_1 U_{t+3-1}^2 + \beta_1 S_{t+3-1}^2$$

$$= \alpha_0 + \alpha_1 U_{t+2}^2 + \beta_1 S_{t+2}^2$$

$$= 0.0172 + 0.1251 U_{t+2}^2 + 0.8611 S_{t+2}^2$$

$$\checkmark = 0.0172 + 0.1251 U_{t+2}^2 + 0.8611 S_{t+2}^2$$