

Set theory assignment question 4

Name: Enter your name here

Acknowledgements: I received no help from other sources on this assignment question (or: I received help from XX with question Y part Z...)

Due: Monday, January 22nd @ 11:20am.

4. In this question, you'll see that matrices and matrix multiplication have the same sorts of properties that functions and function composition have. (If you need a quick introduction or review of matrices and matrix multiplication, see the file labelled "matrix multiplication review".

- (a) Recall that if A is an $m \times n$ matrix and B is an $n \times p$ matrix, then by matrix multiplication we get a $m \times p$ matrix AB . (Note how similar this is to function composition, where given a function $f : A \rightarrow B$ and $g : B \rightarrow C$, we get a new function $fg : A \rightarrow C$!). Prove in a particular case that this operation is **associative**: prove that if A and B are arbitrary 2×2 matrices and C is an arbitrary 2×1 matrix:

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \quad C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

then $(AB)C$ equals $A(BC)$. (That is, prove that if you first multiply A by B , then multiply that result by C , you get the same thing if you first multiply B by C , then multiply that by A).

Solution:

- (b) Prove that this operation is **unital**: that is, for every natural number n , give a specific $n \times n$ matrix E_n so that for any $n \times m$ matrix A ,

$$E_n A = A E_m = A.$$

As always, be sure to prove your claim.

Solution: