

Differential Equations and Numerical Methods

Assignment 1

October 11, 2019

Please use this document as the cover for your assignment. Include a signed statement that it is your own work.

Differential Equation

The following ordinary differential equation is to be solved using numerical methods.

$$\frac{dx}{dt} + \beta x = A e^{-\alpha t}, \text{ where } A, \alpha, \beta > 0 \text{ and } x = x_0 \text{ at } t = 0.$$

It is to be solved from $t = 0$ to $t = 50.0$. It has analytical solution

$$x(t) = A_1 e^{-\alpha t} + A_2 e^{-\beta t}, \text{ where } A_1 = \frac{A}{\beta - \alpha} \text{ and } A_2 = x_0 - A_1.$$

Questions

Answer the questions given here.

1. Rewrite the differential equation you have been given at the start of this document in the correct form for applying the Euler and Euler-Cauchy numerical schemes. Write down an appropriate

Euler method recursive scheme to solve this differential equation for the following values of the parameters and initial conditions:

$$\alpha = 0.2, \beta = 0.5, A = 15.0, x_0 = 7.0.$$

2. For this recursive scheme, draw out as much information as you can on the stability of the scheme from its defining equations, including any restrictions on the values of n for which the scheme is stable.
3. Write a C program to implement the Euler Scheme.
4. Using the result of step (c), calculate the estimates x_n for x for the range of t indicated using a step size of $h = 0.1$. Repeat this for the step size $h = 0.05$.
5. Rewrite your recursive scheme as an Euler-Cauchy scheme for this differential equation. Repeat the previous three steps using this scheme.
6. Compare the values x_n to the correct value $x(t_n)$ on the interval for t for both of the two versions of the numerical scheme and for both step sizes. Comment on your result.