

Differential Equations and Numerical Methods

Assignment 2

November 29, 2019

Please use this document as the cover for your assignment. Include a signed statement that it is your own work.

Differential Equation

The electrostatic potential in a rectangular area with no charge is governed by the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 4[10 - (x + y)].$$

The boundary conditions on the rectangular perimeter are:

$$\phi = 0 \text{ when } x = 0 \text{ or } x = 2.5,$$

$$\phi = 10 \text{ when } y = 0 \text{ or } y = 3.0.$$

Questions

Answer the questions given here.

1. The approximation for the second derivative with respect to x is given here:

$$\frac{\partial^2 \phi}{\partial x^2}(x, y) \approx \frac{\phi(x + h, y) + \phi(x - h, y) - 2\phi(x, y)}{h^2}.$$

Rewrite the Poisson equation you have been given at the start of this document as a difference equation, using this approximation and the corresponding one for y .

2. Rewrite this difference equation in terms of the values

$$P_{i,j} = \phi(ih, jh),$$

on the mesh lattice.

3. Write out a set of equations for the interior points on the lattice, including information on the Boundary conditions.
4. Write these equations in matrix form.
5. Find the solution to this matrix equation, explaining your choice of method for the solution and the implications of your choice for this computation.
6. Write your solution, that is, the values for the $P_{i,j}$, in the form of a set of values on your mesh lattice.