Deadline: 17:00, 26 August 2020

Answer **ALL** questions
Questions carry equal marks. Pass mark: 50 %

Question 1: A risk averse agent, whose utility is given by $U(x) = \ln x$ and whose wealth is $50,000$ is faced with a potential loss of $10,000$ with a probability of 0.1. What is the maximum premium he would be willing to pay to protect himself against this loss? What is the minimum premium that an insurer, with the same utility function and wealth $1,000,000$ will be willing to charge to cover this loss? Explain the difference beteen the two figures.

Question 2: Consider a loss function X with the following probability distribution:

$$P[X = -2] = 0.7, \; P[X = 1] = 0.2, \; P[X = 2] = 0.1$$

Find $\mathrm{VaR}_{0.8}(X)$ and $\mathrm{CVaR}_{0.8}(X)$.

Question 3: A multiple-choice test consists of 5 questions each having 4 possible answers. All questions are equally weighted. The pass mark of the test is 50%. A student gives answers completely at random. Find the probability that she passes the test.

Question 4: An initial payment of £10 yields returns of £5 and £6 at the end of the first and second period respectively. The two periods have equal length. Find the rate of return of the cash stream per period.

Question 5: The mean and covariance matrix of a vector of returns of two assets is

$$\mu = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

respectively. Consider a portfolio of the two assets with weights w_1 and w_2 which satisfy $w_1 + w_2 = 1$, $w_1 \geq 0$ and $w_2 \geq 0$ (no short selling). Find the minimum portfolio variance achievable and the expected portfolio return.

Question 6: Consider the following two investments. Investment A costs £p upfront and B costs £1 upfront. If the economy performs well A brings in £2 but if it performs poorly it makes a loss of £1. The corresponding figures for investment B are a gain of £2 and a loss of £0.5, respectively. There is 50% chance that the economy performs well and 50% chance that it performs poorly. Assume that the company is risk-neutral. Find the value of p (in £) for which the two investments are equivalent.

Question 7: Consider again Question 6 above. Now assume that the company is risk-averse with a utility function $U(x) = 1 - e^{-x}$ where x is the return of the investment. Find the new value of p for which the two investments are equivalent.

Question 8: Consider a risky asset with expected return equal to 2 and variance 4 combined with a risk-free asset with expected return 1. Find the expected-return/variance pairs of all portfolios obtained by combining the two assets if no short selling is allowed. Find also the minimum variance portfolio with expected return not lower than 1.5.

Question 9: Let X be a random variable uniformly distributed between 0 and 1. Let also $Y = \min(X, a)$ where a is a real number such that $0 < a < 1$. Find the expected value and variance of X and Y. Assuming that you are risk-neutral, find the maximum price you would be prepared to pay to insure an item with probability of value loss due to damage Y and insurance excess a.

Question 10:
Consider a loss function $f(x, \xi)$, where x is a fixed decision variable and ξ is a random variable. Show that if f is convex in ξ, then $\mathbb{E}[f(x, \xi)] \geq f(x, \mathbb{E}[\xi])$. You may assume that ξ is discretely distributed with two scenarios. Verify your answer if $f(x, \xi) = (x - \xi)^2$. Give an economic interpretation of the result.

The End