

1

Let $f(x) = x^5 - 5x$

(a) $f'(x) = 5x^4 - 5$

critical values $\frac{dy}{dx} = 0$

$$5x^4 - 5 = 0$$

$$5x^4 = 5$$

$$x^4 = 1$$

$$x = 1 \text{ or } -1$$

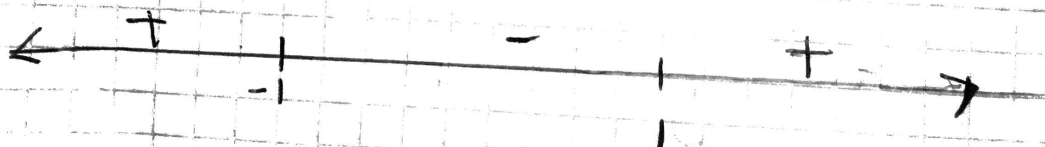
For $x = 1$, $f(x) = (1)^5 - 5(1) = -4$

For $x = -1$, $f(x) = (-1)^5 - 5(-1) = -1 + 5 = 4$

$(1, -4), (-1, 4)$

(b) A function is increasing if its slope is positive and decreases when its slope is negative.
 $x = -1$ or 1

$$f'(x) = 5x^4 - 5 = 0$$



* For all values less than -1 the slope is +ve

* For all values $(-1, 1)$ the slope is -ve

* For values greater than 1 the slope is also positive (+ve)

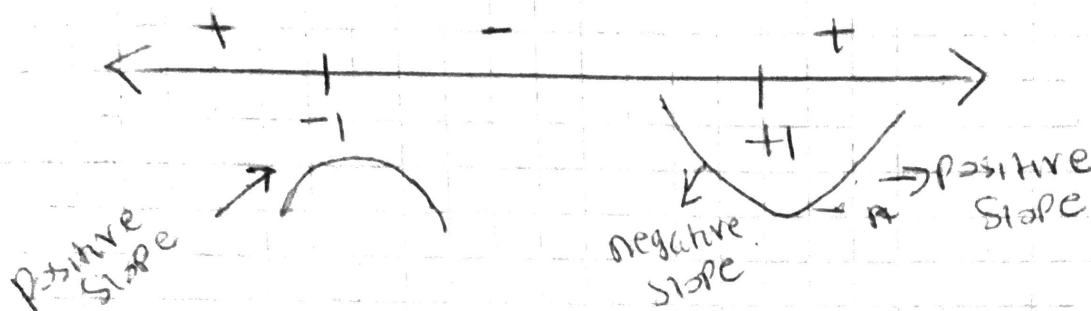
✓ The function is increasing for values;

Inc'd $(-\infty, -1)$ and $(1, \infty)$

✓ Decreasing when; Dec'd $(-1, 1)$

} Ans

Local maximum and local minimum



Local Maximum $(-1, 4)$

Local minimum $(1, -4)$

d For concavity, we find the 2nd derivative and equate to zero

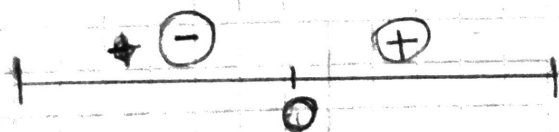
$$f'(x) = 5x^4 - 5$$

$$f''(x) = 20x^3$$

$$20x^3 = 0 \Rightarrow x^3 = \frac{0}{20} = 0$$

$$x^3 = 0$$

$$x = 0$$



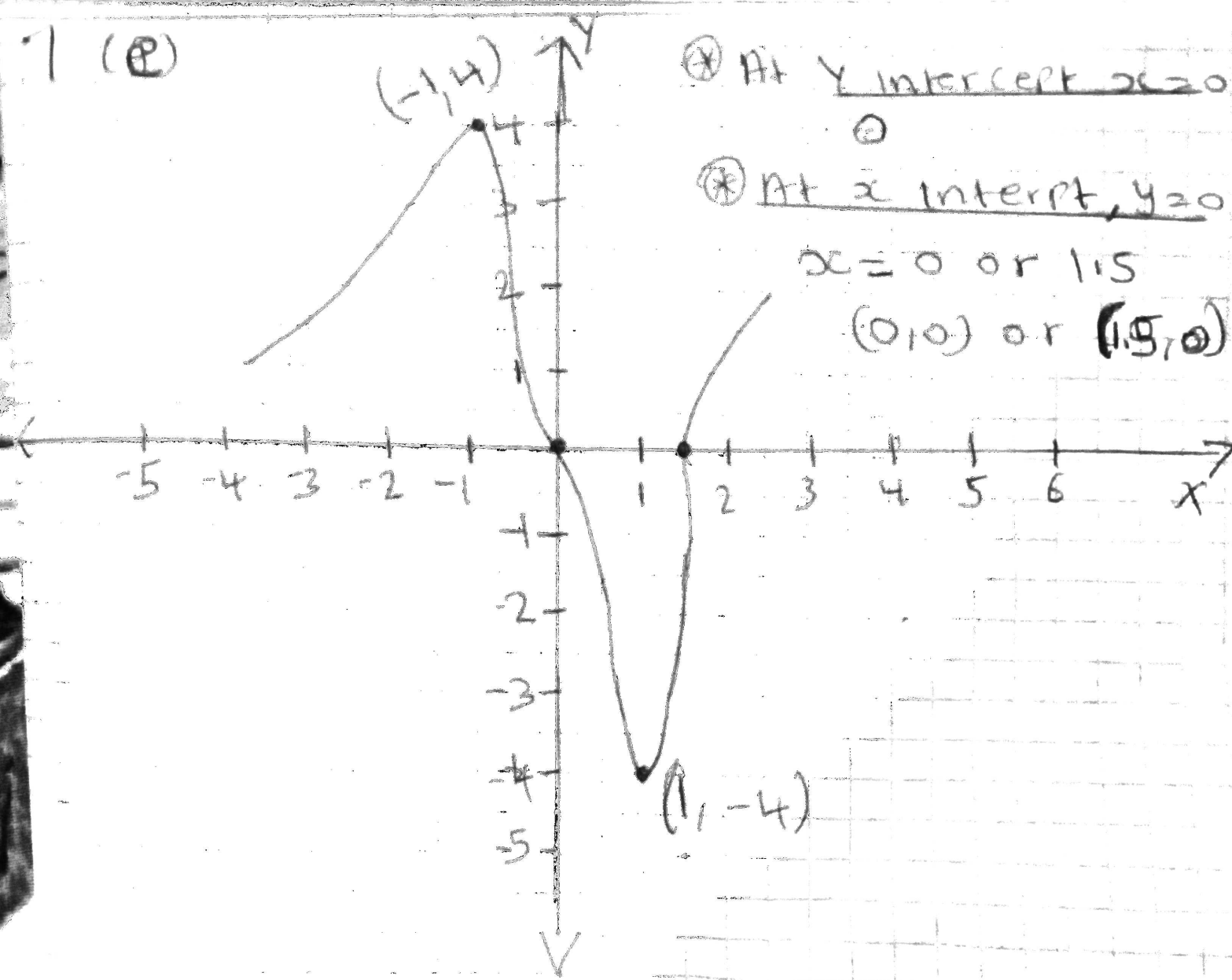
- For all values less than 0 ($x < 0$) we have concave down

For all values more than 0 ($x > 0$) we have concave up

Concave down $(-\infty, 0)$

Concave up $(0, \infty)$

1 (P)



⊗ At Y intercept $x=0$

⊗ At x intercept, $y=0$

$x=0$ or 1.5
 $(0,0)$ or $(1.5,0)$

2

$$g(x) = \frac{x^3}{x^2+1}$$

* Vertical asymptotes occurs at those values of x for which denominator is equal to zero

At $x = -1$ then $g(x)$ will have a vertical asymptotes

ALSO At $x = -1$ If we solve for x

$$x^3 + 1 = 0$$

$$x^3 = -1$$

$$x^3 = \sqrt[3]{-1}$$

✓ Since $\sqrt[3]{-1}$ is not a real number, the graph will have no vertical asymptotes

For the horizontal asymptotes of 'g'

✓ We note that the degree of the numerator is 3 and the degree of the denominator is 3

✓ The graph will have a horizontal asymptote at $y = \frac{1}{1} = 1$
 $y = 1$

c)

Critical values of g

$$g(x) = \frac{x^3}{x^3+1} \Rightarrow \text{use quotient rule}$$

$$\frac{f'(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$= \frac{(x^3+1)3x^2 - x^3(3x^2)}{(x^3+1)^2}$$

$g'(x) = 0$ we get critical values

$$\frac{3x^2 + 3x^2 - 3x^5}{(x^3+1)^2} = 0$$

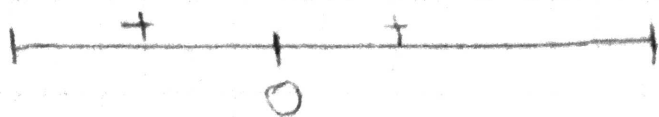
$$\frac{3x^2}{(x^3+1)} = 0 \Rightarrow 3x^2 = 0$$

$x = 0 \Rightarrow \text{Substitute}$

$$g(x) = \frac{0^3}{0^3+1} = 0$$

(d)

⑤

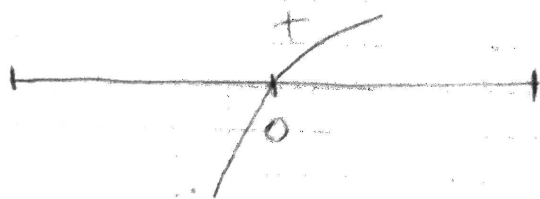


x	
-1	∞
2	+
-2	+
-8	+

Increase $(-\infty, 0)$ & $(0, \infty)$
values

(e)

Local max/min values of g .



Local maximum $(0, 0)$

$$g(x) = \frac{x^3}{x^3+1} = \frac{0}{0+1} = 0$$

No minimum values

(f)

$$g(x) = \frac{x^3}{x^3+1}$$

At x intercept, $y=0$

$$\frac{x^3}{x^3+1} = 0 \Rightarrow x^3 = 0$$

$$x^3 = 0$$

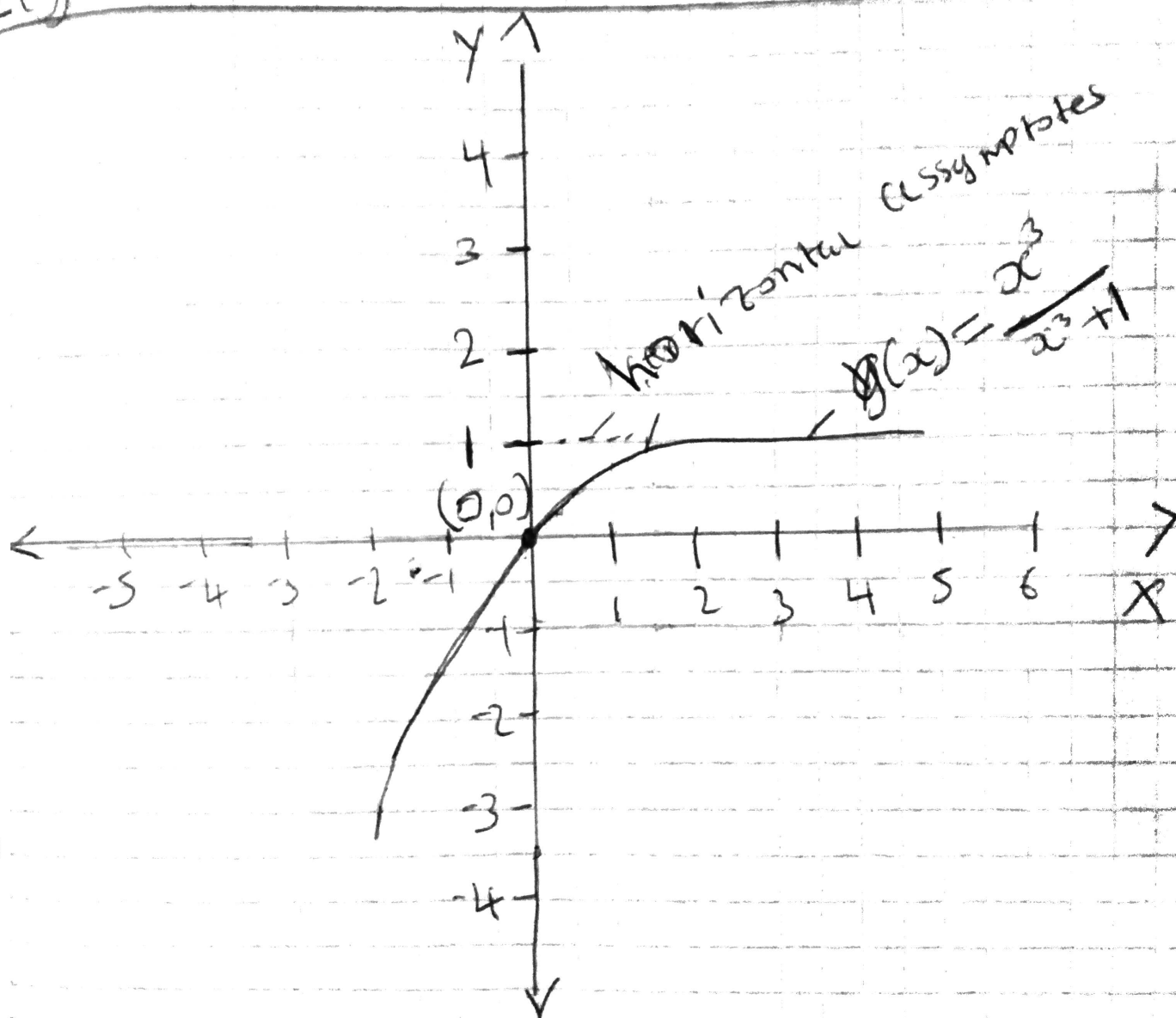
$$x = \underline{\underline{0}}$$

At y intercept, $x=0$

$$g(x) = \frac{0^3}{0^3+1} = \frac{0}{1} = 0$$

$(0, 0)$

2(g)



3) (a) $h(x) = \frac{x-1}{x^2}$

Vertical Asymptotes

$$x^2 = 0$$

$x = 0 \Rightarrow$ is the Vertical Asymptote

2) The degree of the numerator is 1 while the degree of denominator is 2.

Since the degree of ~~of~~ is larger in the denominator, the graph will have a horizontal asymptote at $y = 0$ i.e. x -axis

3) $\frac{x-1}{x^2} = h(x)$

$$h'(x) = \frac{x^2(1) - (x-1)2x}{(x^2)^2} = \frac{x^2 - [2x^2 - 2x]}{[x^2]^2}$$

$$= \frac{x^2 - 2x^2 + 2x}{[x^2]^2} = \frac{-x^2 + 2x}{[x^2]^2}$$

5 (c)

$$h'(x) = 0$$

$$\frac{-x^2 + 2x}{(x^2)^2} = 0 \Rightarrow -x^2 + 2x = 0$$

$$x[-x + 2] = 0$$

$$x = 0 \quad \text{--- (i)}$$

$$-x + 2 = 0$$

$$-x = -2$$

$$x = 2$$

$$\text{At } x = 0$$

$$h(x) = \frac{0-1}{0} = \infty$$

$$\text{At } x = 2$$

$$h(x) = \frac{x-1}{x^2} = \frac{2-1}{(2)^2} = \frac{1}{4} = 0.25$$

Critical Points, $(0, \infty)$, $(2, 0.25)$

3 (d)



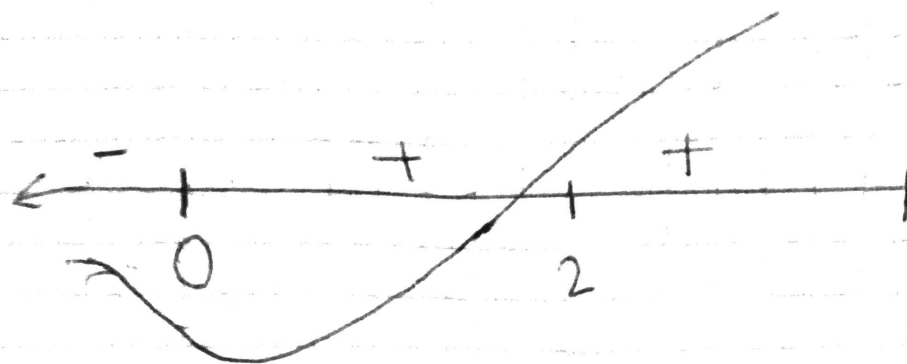
x	Slope
-1	-
0	0 (+)
2	+
3	+

Decreasing Point $(-\infty, 0)$

Increasing Points $(0, 2) \cup (2, \infty)$

$(0, 2)$ and $(2, \infty)$

3 (c)



Maximum values ~~$(0, \infty)$~~ $(2, \frac{1}{4})$

Minimum values ~~$(2, \infty)$~~ $(0, \infty)$

==

3 (d)

x and y intercepts

x -intercepts $y=0$

$$h(x) = \frac{x-1}{x^2}$$

$$\frac{x-1}{x^2} = 0$$

$$x-1 = 0$$

$$x = 1$$

y -intercept $x=0$

$$h(x) = \frac{0-1}{0^2} = -\frac{1}{0} = \infty$$

$$(1, \infty)$$

3 (g)

⑨

