

1. Find the derivatives of the following functions.

a) $y = 5\sin(2x)$

Soln:

$$\frac{d}{dx}(5\sin(2x))$$

$$= 5 \frac{d}{dx}(\sin(2x))$$

Apply the chain rule $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$= 5 \frac{d}{du}(\sin(u)) \frac{d}{dx}(2x)$$

$$\frac{d}{du}(\sin(u)) = \cos(u)$$

$$\frac{d}{dx}(2x) = 2 \frac{d}{dx}(x)$$

$$= 2$$

$$= 5\cos(u) \cdot 2$$

$$= 5\cos(2x) \cdot 2$$

$$= 10\cos(2x)$$

b) $y = -10\cos(x^2)$

Soln:

$$\frac{d}{dx}(-10\cos(x^2))$$

$$= -10 \frac{d}{dx}(\cos(x^2))$$

Apply the chain rule $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$= -10 \frac{d}{du}(\cos(u)) \frac{d}{dx}(x^2)$$

$$\frac{d}{du}(\cos(u)) = -\sin(u)$$

$$\frac{d}{dx}(x^2) = 2x^{2-1}$$

$$= 2x$$

$$= -10(-\sin(u)) \cdot 2x$$

$$= -10(-\sin(x^2)) \cdot 2x$$

$$= 20x\sin(x^2)$$

c) $y = 4x\tan(2x^3)$

Soln:

$$\frac{d}{dx} (4x \tan(2x^3))$$

$$= 4 \frac{d}{dx} (x \tan(2x^3))$$

Apply the product Rule: $(f' \cdot g)' = f' \cdot g + f \cdot g'$

$$= 4 \left(\frac{d}{dx}(x) \tan(2x^3) + \frac{d}{dx}(\tan(2x^3))x \right)$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\tan(2x^3)) = \frac{d}{du}(\tan(u)) \frac{d}{dx}(2x^3)$$

$$= \sec^2(u) \cdot 6x^2$$

$$= 4(1 \cdot \tan(2x^3) + \sec^2(2x^3) \cdot 6x^2x)$$

$$= 4(\tan(2x^3) + 6x^3 \sec^2(2x^3))$$

d) $y = \sec(3x + 5x^2)$

Soln:

$$\frac{d}{dx}(\sec(3x + 5x^2))$$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$= \frac{d}{du}(\sec(u)) \frac{d}{dx}(3x + 5x^2)$$

$$\frac{d}{du}(\sec(u)) = \sec(u)\tan(u)$$

$$\frac{d}{dx}(3x + 5x^2) = \frac{d}{dx}(3x) + \frac{d}{dx}(5x^2)$$

$$= 3 + 10x$$

$$= \sec(u)\tan(u)(3 + 10x)$$

$$= \sec(3x + 5x^2)\tan(3x + 5x^2)(3 + 10x)$$

$$\text{e) } y = \frac{\cot x}{\sin x}$$

Soln:

$$\frac{d}{dx} \left(\frac{\cot(x)}{\sin(x)} \right)$$

$$= \frac{d}{dx}(\cot(x)\csc(x))$$

Apply the product Rule

$$= \frac{d}{dx}(\cot(x))\csc(x) + \frac{d}{dx}(\csc(x))\cot(x)$$

$$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\csc(x)) = -\cot(x)\csc(x)$$

$$= (-\csc^2(x))\csc(x) + (-\cot(x)\csc(x))\cot(x)$$

$$= \csc(x) - 2\csc^3(x)$$

$$f) y = \tan(\sin(x^2))$$

Soln:

$$\frac{d}{dx}(\tan(\sin(x^2)))$$

Apply the chain rule

$$= \frac{d}{du}(\tan(u)) \frac{d}{dx}(\sin(x^2))$$

$$\frac{d}{du}(\tan(u)) = \sec^2(u)$$

$$\frac{d}{dx}(\sin(x^2)) = \frac{d}{du}(\sin(u)) \frac{d}{dx}(x^2)$$

$$= \cos(u) \cdot 2x$$

$$= \sec^2(u)\cos(x^2) \cdot 2x$$

$$= \sec^2(\sin(x^2))\cos(x^2) \cdot 2x$$

Question (2)

Find $\frac{dy}{dx}$ if $x \cos y + y \cos x = xy$

Soln:

$$\frac{d}{dx}(x \cos(y) + y \cos(x))$$

Apply the sum/Difference rule:

$$= \frac{d}{dx}(x \cos(y)) + \frac{d}{dx}(y \cos(x))$$

$$\frac{d}{dx}(x \cos(y)) = \cos(y) \frac{d}{dx}(x)$$

$$\frac{d}{dx}(y \cos(x)) = y \frac{d}{dx}(\cos(x))$$

$$= -y \sin(x)$$

$$= \cos(y) - y \sin(x)$$

Question (3)

Find the equation of the tangent to the curve $y = 2 \sin x + 3 \cos(2x)$ at $x=2$.

Soln:

$$y(2) = 2 \sin 2 + 3 \cos 4 = 1.165$$

$$\frac{dy}{dx} = 2 \cos x - 6 \sin(2x)$$

$$\text{at } x = 2$$

$$y'(x = 2) = 2 \cos(2) - 6 \cos(4)$$

$$\text{slope } m = 2(\cos(2) - 3 \cos(4))$$

$$m = 3.090$$

$$(y - y_1) = m(x - x_1)$$

$$\text{at } (2, 1.165)$$

$$y - 1.165 = 3.090(x - 2)$$

$$y = 3.090x - 6.18 + 1.165$$

$$\text{tangent line is } y = 3.090x - 5.015$$