

Name:
Score:

Math 441
Quiz 3

Read all directions and answer the questions the best that you can. The quiz contains 50 points and reasonable work is expected to be shown for all problems.

Part 1 (7 points)

Find over and under estimates for the area under the curve $f(x) = x^2 + 1$ from $x = 0$ to $x = 2$ using 4 rectangles.

x	0	0.5	1	1.5	2
$f(x)$	1	$\frac{9}{4}$	2	$\frac{13}{4}$	5

left end points

$$\begin{aligned} A &= \Delta x [f(0) + f(0.5) + f(1) + f(1.5)] \\ &= 0.5 \left[1 + \frac{9}{4} + 2 + \frac{13}{4} \right] = \frac{15}{4} \\ &= \underline{\underline{3.75}} \end{aligned}$$

Right end points ($f(x)$)

$$\begin{aligned} A &= \Delta x [f(0.5) + f(1) + f(1.5) + f(2)] \\ &= 0.5 \left[\frac{9}{4} + 2 + \frac{13}{4} + 5 \right] = \frac{23}{4} \\ &= \underline{\underline{5.75}} \end{aligned}$$

Over estimation is 5.75.

Under estimation is 3.75.

Exact area-

$$A = \int_0^2 x^2 + 1 \, dx$$

$$= \left. \frac{x^3}{3} + x \right|_0^2 = \left(\frac{2^3}{3} + 2 \right) - \left(\frac{0^3}{3} + 0 \right)$$

$$\Rightarrow \left(\frac{8}{3} + 2 \right) - 0 = \frac{14}{3}$$

$$A = \frac{14}{3} \approx \underline{\underline{4.667}}$$

Part 2 (5 points)

Find the exact area under the curve $f(x) = x^2 + 1$ from $x = 0$ to $x = 2$.

Solution-

$$\int_0^2 (x^2 + 1) \, dx$$

$$\Rightarrow \int_0^2 x^2 \, dx + \int_0^2 1 \, dx$$

$$\Rightarrow \int_0^2 x^2 \, dx = \left[\frac{x^{2+1}}{2+1} \right]_0^2$$

$$= \left[\frac{x^3}{3} \right]_0^2$$

$$\Rightarrow \left(\frac{2^3}{3} \right) - \left(\frac{0^3}{3} \right) = \underline{\underline{\frac{8}{3}}}$$

$$\int_0^2 1 \, dx = [1 \cdot x]_0^2 = [x]_0^2$$

$$\Rightarrow 2 - 0 = \underline{\underline{2}}$$

$$\Rightarrow \frac{8}{3} + 2 = \frac{8+6}{3} = \underline{\underline{\frac{14}{3}}}$$

Part 3 (2 points each)

Use the graph of $g(x)$ to answer the following.

$\int_{-1}^2 g(x) dx$	$\int_{-1}^2 g(x) dx$ $= \alpha \int_{-1}^2 g dg$ $= \alpha \left[\frac{g^{1+1}}{1+1} \right]_{-1}^2$	$\Rightarrow \alpha \left[\frac{g^2}{2} \right]_{-1}^2 = \alpha \left[\frac{2^2}{2} - \frac{(-1)^2}{2} \right]$ $\Rightarrow \alpha \left[\frac{4}{2} - \frac{1}{2} \right] = \underline{\underline{\frac{3}{2}\alpha}}$
$\int_2^4 g(x) dx$	$\int_2^4 g(x) dx$ $\Rightarrow \alpha \int_2^4 g dg$	$\Rightarrow \alpha \left[\frac{g^{1+1}}{1+1} \right]_2^4 = \alpha \left[\frac{4^2}{2} - \frac{2^2}{2} \right]$ $= \alpha \left(\frac{16}{2} - \frac{4}{2} \right) = \frac{12}{2}\alpha$ $= \underline{\underline{6\alpha}}$
$\int_{-1}^4 g(x) dx$	$\int_{-1}^4 g(x) dx = \alpha \int_{-1}^4 g dg$	$\alpha \left[\frac{g^{1+1}}{1+1} \right]_{-1}^4 = \alpha \left[\frac{4^2}{2} - \frac{(-1)^2}{2} \right]$ $= \frac{16}{2}\alpha - \frac{1}{2}\alpha = \underline{\underline{\frac{15}{2}\alpha}}$

Part 4 (5 points each)

Evaluate all of the following integrals.

1.) $\int x^{4/7} - 4x^{-2} - 4\sec^2 x \, dx$

$\Rightarrow \int x^{4/7} dx - \int 4x^{-2} dx - \int 4\sec^2 x \, dx$ $\Rightarrow \int x^{4/7} dx \Rightarrow \frac{x^{4/7+1}}{4/7+1} \Rightarrow \frac{7}{11} x^{11/7}$ $\Rightarrow \int 4x^{-2} dx = 4 \int x^{-2} dx$	$4 \cdot \frac{x^{-2+1}}{-2+1} = 4(x^{-1})$ $\Rightarrow -4x^{-1} \Rightarrow -\frac{4}{x}$ $\int 4\sec^2 x \, dx = \tan x$ $\Rightarrow \frac{7}{11} x^{11/7} + \frac{4}{x} - \tan x$	$\Rightarrow \frac{7}{11} x^{11/7} + \frac{4}{x} - \tan x + C$
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2.) $\int 3\sin \theta - \csc \theta \cot \theta \, d\theta$

$\Rightarrow \int 3\sin \theta \, d\theta - \int \csc \theta \cot \theta \, d\theta$ $\int 3\sin \theta \, d\theta = 3 \cdot \int \sin \theta \, d\theta$ $= 3 \cdot (-\cos(\theta))$ $= -3\cos \theta$ $\Rightarrow \int \csc \theta \cot \theta \, d\theta$ using substitution rule: let $u = \csc \theta$	$\Rightarrow \int \csc \theta \cot \theta \, d\theta = -\csc \theta$ $\therefore \int -1 \, du \Rightarrow (-1)u$ $\Rightarrow (-1)\csc(\theta)$ $= -\csc \theta$ $\Rightarrow -3\cos \theta - (-\csc \theta)$ $= -3\cos \theta + \csc \theta + C$
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$$3.) \int \sqrt[3]{x^5} + \frac{6}{x^4} dx$$

$$\Rightarrow \int \sqrt[3]{x^5} dx + \int \frac{6}{x^4} dx$$

$$\int \sqrt[3]{x^5} dx = x^{\frac{5}{3}} \quad x > 0$$

$$\int x^{\frac{5}{3}} = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1}$$

$$\Rightarrow \frac{x^{\frac{8}{3}}}{\frac{8}{3}}$$

$$4.) \int_2^3 x^2(4x-3) dx$$

$$\Rightarrow \int_2^3 4x^3 - 3x^2 dx$$

$$= \int_2^3 4x^3 dx - \int_2^3 3x^2 dx$$

$$= \left[\frac{4x^4}{4} - \frac{3x^3}{3} \right]_2^3$$

$$5.) \int 8x^3(x^4+2)^5 dx$$

$$\Rightarrow 8 \int x^3(x^4+2)^5 dx$$

$$\text{let } t = x^4 + 2$$

$$\Rightarrow 8 \cdot \int \frac{t^5}{4} dt$$

$$\Rightarrow 8 \times \frac{1}{4} \int t^5 dt$$

$$\Rightarrow \frac{8}{4} \left[\frac{t^{5+1}}{5+1} \right]$$

$$\Rightarrow \frac{8}{4} \times \left[\frac{x^4+2}{5+1} \right]^{5+1}$$

$$\Rightarrow \frac{x^{\frac{8}{3}} \times 3}{8}$$

$$= \frac{3}{8} x^{\frac{8}{3}}$$

$$\Rightarrow \int \frac{6}{x^4} dx = 6 \int \frac{1}{x^4} dx$$

$$= \frac{1}{x^4} = x^{-4}$$

$$\Rightarrow 6 \cdot \int x^{-4} dx$$

$$\Rightarrow 6 \cdot \frac{x^{-4+1}}{-4+1} = \frac{x^{-3}}{-3}$$

$$\Rightarrow 6 \cdot \left(-\frac{1}{3x^3} \right)$$

$$= \frac{-1 \cdot 6}{x^3 \times 3} = -\frac{2}{x^3}$$

$$\Rightarrow \frac{3}{8} x^{\frac{8}{3}} - \frac{2}{x^3} + C$$

$$\Rightarrow x^4 - x^3 \Big|_2^3$$

$$\Rightarrow (3^4 - 3^3) - (2^4 - 2^3)$$

$$\Rightarrow (3^4 - 3^3) - (2^4 - 2^3)$$

$$= 54 - 8$$

$$= \underline{\underline{46}}$$

$$\Rightarrow \frac{8}{4} \left(\frac{x^4+2}{5+1} \right)^{5+1}$$

$$\Rightarrow 8 \times \frac{1}{4} \frac{(x^4+2)^6}{6}$$

$$= \frac{8}{4} \times \frac{(x^4+2)^6}{6}$$

$$= \frac{8}{24} (x^4+2)^6$$

$$\Rightarrow \frac{1}{3} \left(\frac{x^4+2}{3} \right)^6$$

$$= \frac{1}{3} (x^4+2)^6$$

$$\Rightarrow \underline{\underline{\frac{1}{3} (x^4+2)^6 + C}}$$

Part 5 (7 points)

Find $h(x)$ given $h'(x) = 5x^4 - 12x^2 + 6 - 4\sin x$ and $h(0) = 3$

$$\stackrel{\text{881n}}{=} \int_0^3 (5x^4 - 12x^2 + 6 - 4\sin x) dx \Rightarrow 243 + 108 + 18 - 4(-\cos 3 + 1)$$

$$\Rightarrow \int_0^3 5x^4 dx - \int_0^3 12x^2 dx + \int_0^3 6 dx - \int_0^3 4\sin x dx \Rightarrow \underline{\underline{4\cos 3 + 149}}$$

$$\Rightarrow 5 \cdot \int_0^3 x^4 dx = 5 \left[\frac{x^{4+1}}{4+1} \right]_0^3$$

$$= 5 \left[\frac{x^5}{5} \right]_0^3 = 5 \left[\frac{3^5}{5} - 0 \right]$$

$$= 5 \times \frac{243}{5} = \underline{\underline{243}}$$

$$\Rightarrow \int_0^3 12x^2 dx = 12 \int_0^3 x^2 dx$$

$$\Rightarrow 12 \left[\frac{x^{2+1}}{2+1} \right]_0^3 = 12 \left[\frac{x^3}{3} \right]_0^3$$

$$\Rightarrow 12 \left[\frac{3^3}{3} - 0 \right] = 12 \cdot \frac{27}{3}$$
$$= \underline{\underline{108}}$$

$$\Rightarrow \int_0^3 6 dx = [6x]_0^3$$

$$= (6 \times 3) - 0 = \underline{\underline{18}}$$

$$\Rightarrow \int_0^3 4\sin(x) dx = 4(-\cos(3) + 1)$$

$$\Rightarrow 4 \cdot \int_0^3 \sin(x) dx$$

$$\Rightarrow 4 \left[-\cos(x) \right]_0^3$$

$$= -4 \cos 3 + 4$$