

### Question 1.

(i) If the graph is pictured of  $f''(x)$

$f''(x)$  is zero at  $x=1$ ,  $x=4$  and  $x=7$

$f''(x)$  is negative before  $x=1$  and positive after  $x=1$ . at  $x=1$   
 $f''(x)$  is defined

$f''(x)$  is positive before  $x=7$  and negative after  $x=7$ . At  $x=7$   
 $f''(x)$  is defined

Conclusion:  $x=1$  and  $x=7$  are inflection points.

(ii) If the graph is pictured of  $f'(x)$ .

The derivative of the graph equals zero at  $x=2$ ,  $x=4$ ,  $x=6$ .

$x$ -Value	2	4	6
L.H sign of $f''(x)$	-	-	+
R.H sign of $f''(x)$	+	+	-

Therefore inflection points are:  $x=2$ ,  $x=4$ ,  $x=6$

## Question 2

An inflection point is where the concavity of a graph changes. In our case, the trajectory (trend) of bitcoin price will change after six days. If the price was increasing at an increasing rate, it will start to increase at a decreasing rate.

## Question 3

$$f(x) = \frac{1}{4}x^4 - x^3$$

(a) Domain :  $(-\infty, \infty)$

$$\frac{1}{4}x^4 - x^3 = 0$$

$$\frac{1}{4}x^4 = x^3 \quad ; x=0$$

$$\frac{1}{4}x = 1$$
$$x = 4$$

Zeros:  $x=0, x=4$

- $\Rightarrow$  No vertical asymptotes ; No point where denominator equals 0
- $\Rightarrow$  No horizontal asymptotes
- $\Rightarrow$  No oblique asymptotes ; The degree of the numerator is not one degree greater than the denominator

$$(b) f'(x) = x^3 - 3x^2$$

$$x^3 - 3x^2 = 0 \quad ; \quad x = 0$$

$$x^3 = 3x^2$$

$$x = 3$$

Critical Points:  $x = 0, x = 3$

	$(-\infty, 0)$	0	$(0, 3)$	3	$(3, \infty)$
$f'(x)$	-	0	-	0	+

Intervals of Increase:  $(3, \infty)$

Intervals of Decrease:  $(-\infty, 0) \cup (0, 3)$

$$f(0) = 0$$

$$f(3) = \frac{1}{4}(3^4) - 3^3 = \frac{-27}{4}$$

Local minima:  $(3, \frac{-27}{4})$

$$(c) f''(x) = 3x^2 - 6x$$

$$3x^2 - 6x = 0 \quad ; \quad x = 0$$

$$3x^2 = 6x$$

$$3x = 6$$

$$x = 2$$

$$f(0) = 0$$

$$f(2) = \frac{1}{4}(2^4) - 2^3 = -4$$

	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, \infty)$
$f''(x)$	+		-		+

Concave up:  $(-\infty, 0) \cup (2, \infty)$

Concave down:  $(0, 2)$

Inflection points:  $(0, 0)$  and  $(2, -4)$

(a)

