

Emily Murphy Assignment Assignment 4 - Related Rates and Linearization and Hyperbolic Functions
MATH140, Winter 2020
due 04/04/2020 at 11:59pm EDT.

This is a practice assignment. You have unlimited attempts per question.

1. (1 point)

A price p (in dollars) and demand x for a product are related by

$$2x^2 + 4xp + 50p^2 = 9800.$$

If the price is increasing at a rate of 2 dollars per month when the price is 10 dollars, find the rate of change of the demand.

Rate of change of demand = _____

2. (1 point)

Suppose that $x = x(t)$ and $y = y(t)$ are both functions of t . If

$$y^2 + xy - 3x = -7,$$

and $dy/dt = 4$ when $x = 4$ and $y = -5$, what is dx/dt ?

$dx/dt =$ _____

3. (1 point)

A rock is thrown into a still pond and causes a circular ripple. If the radius of the ripple is increasing at a rate of 4 feet per second, how fast is the circumference changing when the radius is 19 feet?

Change in circumference = _____

4. (1 point) A street light is at the top of a 20 ft tall pole.

A woman 6 ft tall walks away from the pole with a speed of 6 ft/sec along a straight path. How fast is the tip of her shadow moving when she is 35 ft from the base of the pole?

Note: You should draw a picture of a right triangle with the vertical side representing the pole, and the other end of the hypotenuse representing the tip of the woman's shadow. Where does the woman fit into this picture? Label her position as a variable, and label the tip of her shadow as another variable. You might like to use similar triangles to find a relationship between these two variables.

5. (1 point) A spherical snowball is melting in such a way

that its diameter is decreasing at rate of 0.3 cm/min. At what rate is the volume of the snowball decreasing when the diameter is 12 cm. (Note the answer is a positive number).

6. (1 point) A plane flying with a constant speed of 24 km/min passes over a ground radar station at an altitude of 16 km and climbs at an angle of 25 degrees.

At what rate is the distance from the plane to the radar station increasing 3 minutes later?

The distance is increasing at _____ km/min.

7. (1 point) At noon, ship A is 20 nautical miles due west of ship B. Ship A is sailing west at 21 knots and ship B is sailing north at 15 knots. How fast (in knots) is the distance between the ships changing at 6 PM?

The distance is changing at _____ knots.

(Note: 1 knot is a speed of 1 nautical mile per hour.)

8. (1 point) In a simple electric circuit, Ohm's law states that $V = IR$, where V is the voltage in volts, I is the current in amperes, and R is the resistance in ohms. Assume that, as the battery wears out, the voltage decreases at 0.04 volts per second and, as the resistor heats up, the resistance is increasing at 0.01 ohms per second. When the resistance is 100 ohms and the current is 0.01 amperes, at what rate is the current changing?

_____ amperes per second

9. (1 point) Use linear approximation, i.e. the tangent line, to approximate 4.8^6 as follows:

Let $f(x) = x^6$. The equation of the tangent line to $f(x)$ at $x = 5$ can be written in the form $y = mx + b$

where m is: _____

and where b is: _____

Using this, we find our approximation for 4.8^6 is _____

10. (1 point) Use linear approximation, i.e. the tangent line, to approximate $\sqrt[3]{8.12}$ as follows. Let $f(x) = \sqrt[3]{x}$ and find the equation of the tangent line to $f(x)$ at $x = 8$ in the form $y = mx + b$.

Note: The values of m and b are rational numbers which can be computed by hand. You need to enter expressions which give m and b exactly. You may not have a decimal point in the answers to either of these parts.

$m =$ _____

$b =$ _____

Using these values, find the approximation.

$\sqrt[3]{8.12} \approx$ _____

Note: You can enter decimals for the last part, but it will have to

be entered to very high precision (correct for 6 places past the decimal point).

11. (1 point)

Evaluate the following as a rational number n/m :

$$13 \sinh(\ln 2) = \underline{\hspace{2cm}}$$

12. (1 point) Find the derivative of

$$f(x) = \sinh(x^2 + 3).$$

$$f'(x) = \underline{\hspace{2cm}}$$

13. (1 point) If

$$f(x) = \frac{1 - \cosh x}{9 + \cosh x}$$

$$\text{then } f'(x) = \underline{\hspace{2cm}}.$$

14. (1 point) Find the derivative of

$$f(x) = \sinh(x) \tanh(x).$$

$$f'(x) = \underline{\hspace{2cm}}$$