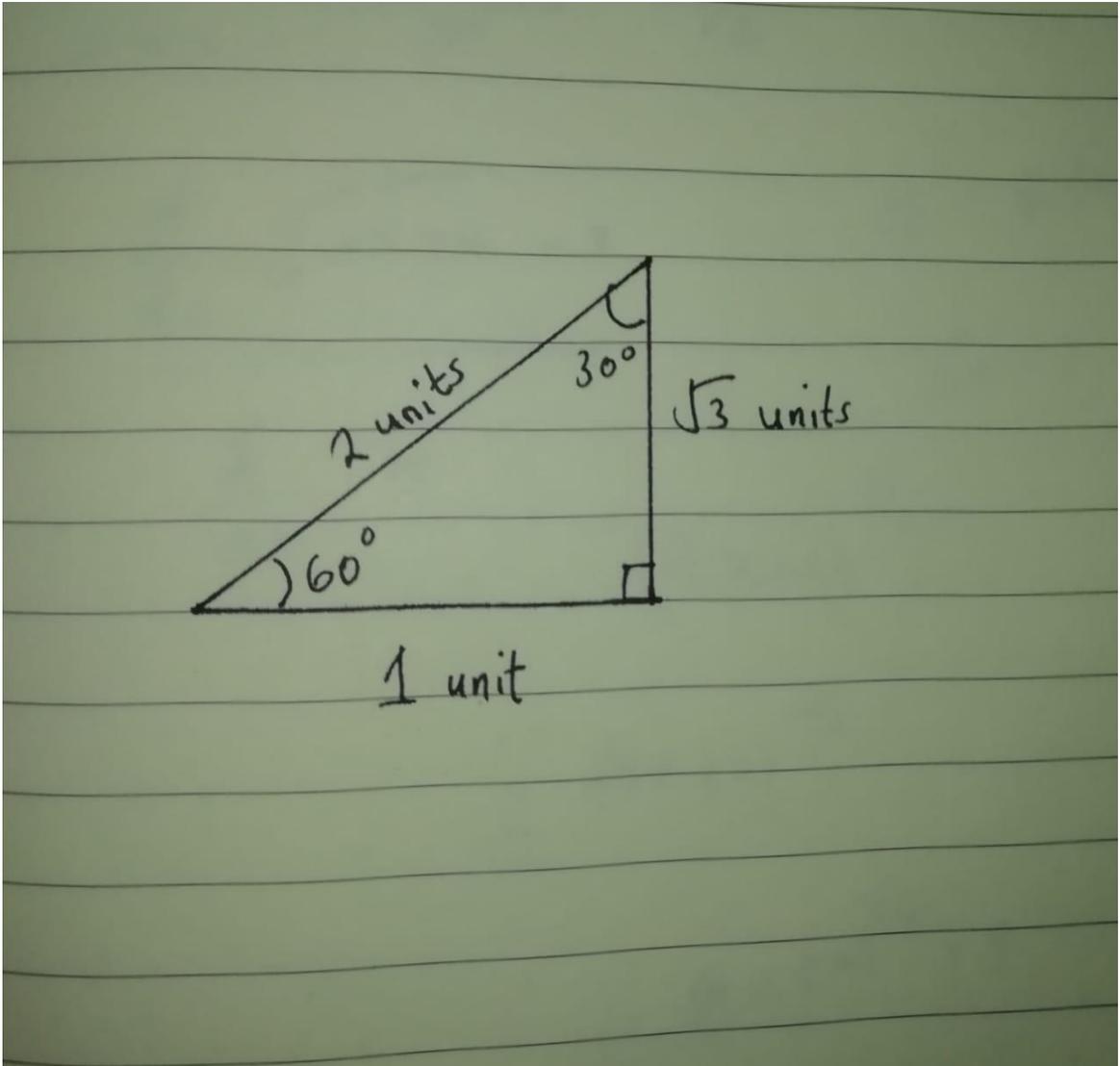1. arctan $\left(\frac{1}{\sqrt{3}}\right) = 30°$

    $\cos(30°) = \frac{\sqrt{3}}{2}$

2. $\lim\limits_{x \to 0} \left(\frac{\sqrt{9x+a}-3}{x}\right) = \frac{3}{2}$

$$\lim_{x \to 0} \left(\frac{\sqrt{9x+a}-3}{x}\right) = \lim_{x \to 0} \left(\frac{\frac{d}{dx}(\sqrt{9x+a}-3)}{\frac{d}{dx}(x)}\right)$$

Differentiating the numerator using chain rule:

$$\frac{d}{dx}\left(\sqrt{9x+a}-3\right)=\frac{9}{2\sqrt{9x+a}}$$

$$\lim_{x\to 0}\left(\frac{\frac{d}{dx}(\sqrt{9x+a}-3)}{\frac{d}{dx}(x)}\right)=\lim_{x\to 0}\left(\frac{9}{2\sqrt{9x+a}}\right)=\frac{9}{2\sqrt{a}}$$

$$\frac{9}{2\sqrt{a}}=\frac{3}{2}$$

$$a=9$$

3. 1st Case:

$$\frac{1}{\sqrt{x^2-1}}$$

$$x^2-1>0$$
$$x^2>1$$
-1 > x > 1

2nd Case:

$$\frac{1}{\sqrt{1-x^2}}$$

$$1-x^2>0$$
$$x^2<1$$
-1 < x < 1

Since the domain of $\sin^{-1}x=[-1,1]$, $\frac{1}{\sqrt{1-x^2}}$ is the correct answer.

4. $\lim\limits_{x\to 0}\left(\frac{\arctan(x^2)}{x\arcsin(x)}\right)$

   Applying L'Hopital's rule:

$$\lim_{x\to 0}\left(\frac{\arctan(x^2)}{x\arcsin(x)}\right)=\lim_{x\to 0}\left(\frac{\frac{d}{dx}(\arctan(x^2))}{\frac{d}{dx}(x\arcsin(x))}\right)$$

$$\frac{d}{dx}(\arctan(U)) = \frac{U'}{U^2+1}$$

$$\frac{d}{dx}(\arcsin(U)) = \frac{U'}{\sqrt{1-U^2}}$$

$$\frac{d}{dx}(\arctan(x^2)) = \frac{2x}{x^4+1}$$

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(x\arcsin(x)) = x\left(\frac{1}{\sqrt{1-x^2}}\right) + \arcsin(x)$$

$$\lim_{x\to 0}\left(\frac{\arctan(x^2)}{x\arcsin(x)}\right) = \lim_{x\to 0}\left(\frac{\frac{2x}{x^4+1}}{(x^4+1)(\frac{x}{\sqrt{1-x^2}} + \arcsin(x))}\right)$$

Simplifying:

$$\frac{2x}{(x^4+1)(\frac{x}{\sqrt{1-x^2}} + \arcsin(x))} = \frac{2x\sqrt{1-x^2}}{(\arcsin(x)\sqrt{1-x^2} + x)((x^4+1))}$$

Applying L'Hopital's rule:

$$\lim_{x\to 0}\left(\frac{2x\sqrt{1-x^2}}{(\arcsin(x)\sqrt{1-x^2}+x)((x^4+1))}\right) = \lim_{x\to 0}\left(\frac{\frac{d}{dx}(2x\sqrt{1-x^2})}{\frac{d}{dx}((\arcsin(x)\sqrt{1-x^2}+x)((x^4+1)))}\right)$$

$$\frac{d}{dx}\left(2x\sqrt{1-x^2}\right) = \frac{2(-2x^2+1)}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}((\arcsin(x)\sqrt{1-x^2}+x)((x^4+1))) = (x^4+1)\frac{d}{dx}((\arcsin(x)\sqrt{1-x^2}+x) + \frac{d}{dx}(x^4+1)(\arcsin(x)\sqrt{1-x^2}+x)$$

$$= \left(\frac{x\arcsin(x)}{\sqrt{1-x^2}} + 2\right)(x^4+1) + (\arcsin(x)\sqrt{1-x^2}+x)(4x^3)$$

$$\frac{d}{dx}\left((\arcsin(x)\sqrt{1-x^2}+x)((x^4+1))\right)$$
$$=\frac{-5x^5\arcsin(x)+4x^3\arcsin(x)+6x^4\sqrt{1-x^2}-x arcsin(x)}{\sqrt{1-x^2}}+2$$

$$\lim_{x\to 0}\left(\frac{2x\sqrt{1-x^2}}{(\arcsin(x)\sqrt{1-x^2}+x)((x^4+1))}\right)$$

$$=\lim_{x\to 0}\left(\frac{\dfrac{2(-2x^2+1)}{\sqrt{1-x^2}}}{\dfrac{-5x^5\arcsin(x)+4x^3\arcsin(x)+6x^4\sqrt{1-x^2}-x arcsin(x)}{\sqrt{1-x^2}}+2}\right)$$

Substituting x with 0:

$$\frac{2(-2*0^2+1)}{\sqrt{1-0^2}}=2$$

$$\frac{-5*0^5\arcsin(0)+4*0^3\arcsin(0)+6*0^4\sqrt{1-0^2}-0*arcsin(0)}{\sqrt{1-0^2}}+2=2$$

$$\lim_{x\to 0}\left(\frac{\arctan(x^2)}{x\arcsin(x)}\right)=\frac{2}{2}=1$$

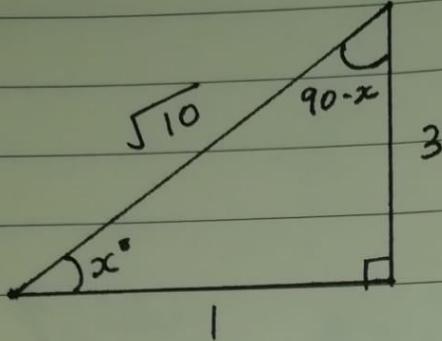5. Sin(arctan(3)+arctan(2)) = sin(arctan(3))*cos(arctan(2)) + cos(arctan(3))*sin(arctan(2))
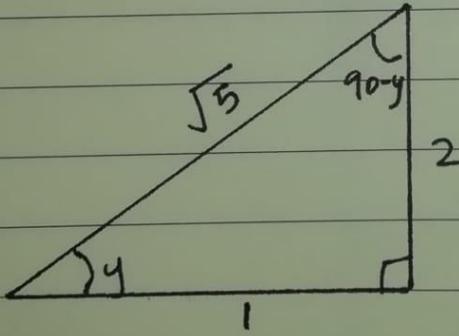
Let arctan(3) = x
tan(x) = 3

Let arctan(2) = y
tan(y)=2

Sin(arctan(3)+arctan(2)) = sin(x)*cos(y) + cos(x)*sin(y)

ay

$\tan(x) = 3$, $\sin(x) = 3/\sqrt{10}$, $\cos(x) = \frac{1}{\sqrt{10}}$

$\tan(y) = 2$

$\sin(y) = \dfrac{2}{\sqrt{5}}$

$\cos(y) = \dfrac{1}{\sqrt{5}}$

$\text{Sin}(\arctan(3)+\arctan(2)) = \dfrac{3}{\sqrt{10}}*\dfrac{1}{\sqrt{5}} + \dfrac{1}{\sqrt{10}}*\dfrac{2}{\sqrt{5}}$

$= \dfrac{\sqrt{2}}{2}$

6. $\lim\limits_{x\to-\infty}\left(x\ln(1+\frac{1}{x})\right) = \lim\limits_{x\to-\infty}\left(\frac{d}{dx}(x\ln(1+\frac{1}{x}))\right)$

Applying L'Hopital's rule:

$$\lim_{x\to-\infty}\left(\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}\right) = \lim_{x\to-\infty}\left(\frac{\frac{d}{dx}(\ln\left(1+\frac{1}{x}\right))}{\frac{d}{dx}(\frac{1}{x})}\right)$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

Solve $\frac{d}{dx}\left(\ln\left(1+\frac{1}{x}\right)\right)$

Applying chain rule:

$$= \frac{1}{1+x}*\frac{d}{dx}\left(1+\frac{1}{x}\right) = \frac{1}{1+x}*\frac{-1}{x^2} = \frac{-1}{x(x+1)}$$

$$\lim_{x\to-\infty}\left(x\ln(1+\frac{1}{x})\right) = \lim_{x\to-\infty}\left(\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}\right)$$

$$\frac{d}{dx}\left(x\ln\left(1+\frac{1}{x}\right)\right) = \frac{x}{x+1}$$

$$\lim_{x\to-\infty}\left(\frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}\right) = \lim_{x\to-\infty}\left(\frac{x}{x+1}\right)$$

Applying L'Hopital's rule:

$$\lim_{x\to-\infty}\left(\frac{x}{x+1}\right) = \lim_{x\to-\infty}\left(\frac{\frac{d}{dx}(x)}{\frac{d}{dx}(x+1)}\right)$$

$$\frac{\frac{d}{dx}(x)}{\frac{d}{dx}(x+1)} = \frac{1}{1}$$

$$\lim_{x\to-\infty}\left(x\ln(1+\frac{1}{x})\right) = 1$$

7. $\int_a^b \frac{e^x}{1+e^{2x}}$

Applying U-substitution:

$u = e^x$

$\frac{du}{dx} = e^x$

$= \int_a^b \frac{e^x}{1+u^2} * \frac{du}{e^x}$

$\int \frac{du}{1 + u^2}$

But $\int \frac{du}{1+u^2} = \arctan(u)$

$=\arctan(e^x)$

$\int_a^b \frac{e^x}{1+e^{2x}} = \arctan(e^x)\Big\{_a^b$

$= \arctan(e^b)- \arctan(e^a)$

8.

$\lim\limits_{x\to a} \left(\frac{\int_a^x f(t)dt}{f(x)}\right) = \lim\limits_{x\to a} \left(\frac{\int_a^a f(t)dt}{f(x)}\right)$

But $\int_a^a f(t)dt = f(a)$

Applying L′Hopital′s rule:
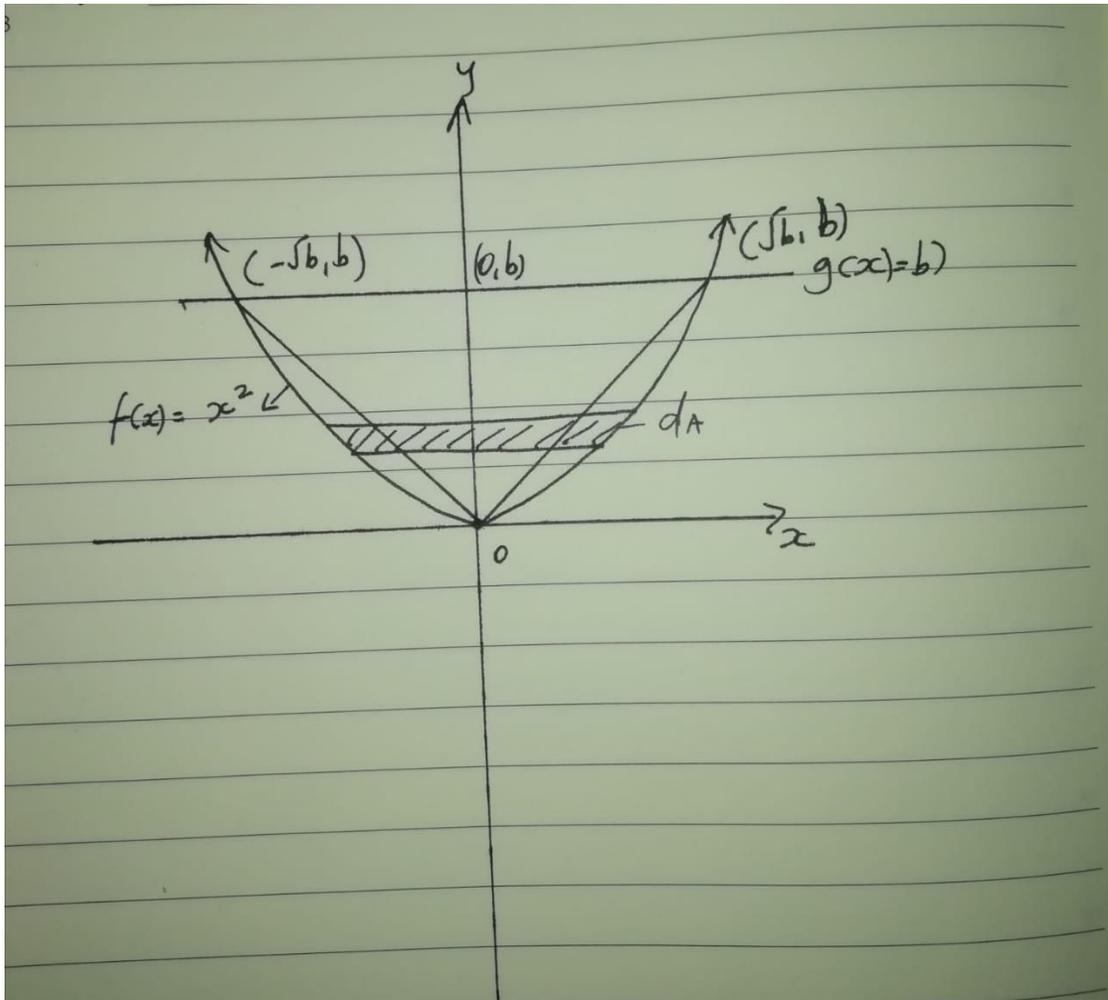
$\lim\limits_{x\to a} \left(\frac{\int_a^a f(t)dt}{f(x)}\right) = \lim\limits_{x\to a} \left(\frac{\frac{d}{dx}(\int f(t)dt)}{\frac{d}{dx}(f(x))}\right)$

$=\lim\limits_{x\to a} \left(\frac{f(x)}{f'(x)}\right)$; f(a) = 0

Thus $\lim\limits_{x\to a} \left(\frac{\int_a^x f(t)dt}{f(x)}\right) = 0$

9. T(b)=$\frac{1}{2}(2\sqrt{b} * b) = b^{\frac{3}{2}}$

$A(b) = 2 \int_0^b \sqrt{y} = 2 * \frac{2}{3} * y^{\frac{3}{2}} \Big\{_0^b$

$= \frac{4}{3} b^{\frac{3}{2}}$

$\lim\limits_{b \to 0} \left(\frac{T(b)}{A(b)} \right) = \lim\limits_{b \to 0} \left(\frac{b^{\frac{3}{2}}}{\frac{4}{3} b^{\frac{3}{2}}} \right)$

$= \frac{3}{4}$

10. $\lim\limits_{x\to\infty}\left(x(a^{\frac{1}{x}}-1)\right)=\lim\limits_{x\to\infty}\left(\dfrac{(a^{\frac{1}{x}}-1)}{\frac{1}{x}}\right)$

Applying L'Hopital's rule:

$$\lim\limits_{x\to\infty}\left(\dfrac{a^{\frac{1}{x}}-1}{\frac{1}{x}}\right)=\lim\limits_{x\to\infty}\left(\dfrac{\frac{d}{dx}(a^{\frac{1}{x}}-1)}{\frac{d}{dx}(\frac{1}{x})}\right)$$

$$\dfrac{\frac{d}{dx}\left(a^{\frac{1}{x}}-1\right)}{\frac{d}{dx}(\frac{1}{x})}=\dfrac{\frac{a^{\frac{1}{x}}\ln(a)}{x^2}}{\frac{1}{x^2}}=a^{\frac{1}{x}}\ln(a)$$

$$\lim\limits_{x\to\infty}\left(a^{\frac{1}{x}}\ln(a)\right)=\ln(a)*\lim\limits_{x\to\infty}\left(a^{\frac{1}{x}}\right)$$

$$\lim\limits_{x\to\infty}\left(a^{\frac{1}{x}}\right)=a^{\lim\limits_{x\to\infty}\left(\frac{1}{x}\right)}$$
$$=a^0=1$$

$$\lim\limits_{x\to\infty}\left(a^{\frac{1}{x}}\ln(a)\right)=\ln(a)*1$$

$$\lim\limits_{x\to\infty}\left(x(a^{\frac{1}{x}}-1)\right)=\ln(a)$$