

Q2 (a) (i)

$$(i) E(T) = E(x) = \lambda$$

$$f(x, \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$E(T) = \frac{e^{-6} 6^3}{3!}$$

Since T = time for the arrival of the third customer

$$= \underline{\underline{0.08924}}$$

$$(ii) P(T) > 1)$$

$$= P(T) > 1 = 1 - P(T \leq 1)$$

$$= 1 - \{P(T=0) + P(T=1)\}$$

$$= 1 - \left\{ \frac{e^{-6} 6^0}{0!} + \frac{e^{-6} 6^1}{1!} \right\}$$

$$= 1 - [2.4788 \times 10^{-3} + 0.01487]$$

$$= 1 - [\quad]$$

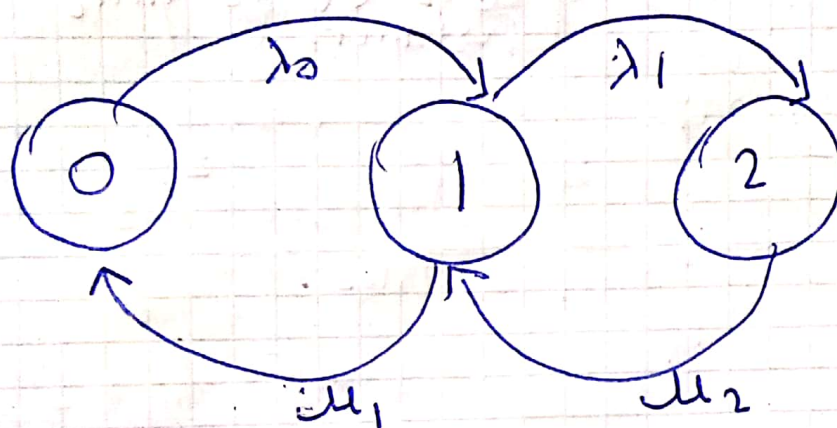
$$= \underline{\underline{0.9826}}$$

Q2 (b)

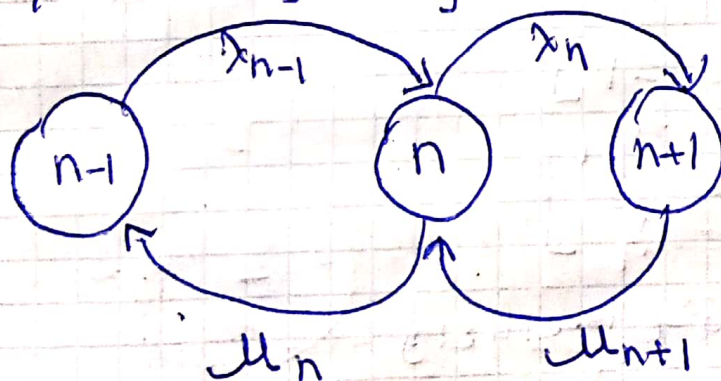
$\lambda_0 = \lambda(2-0) = \lambda_2$
 $\lambda_1 = \lambda(2-1) = \lambda_1$
 $\lambda_2 = \lambda(2-2) = \lambda_0$

$\mu_0 = \mu_0$
 $\mu_2 = \mu_1$
 $\mu_3 = \mu_2$

$k = 0, 1, 2$

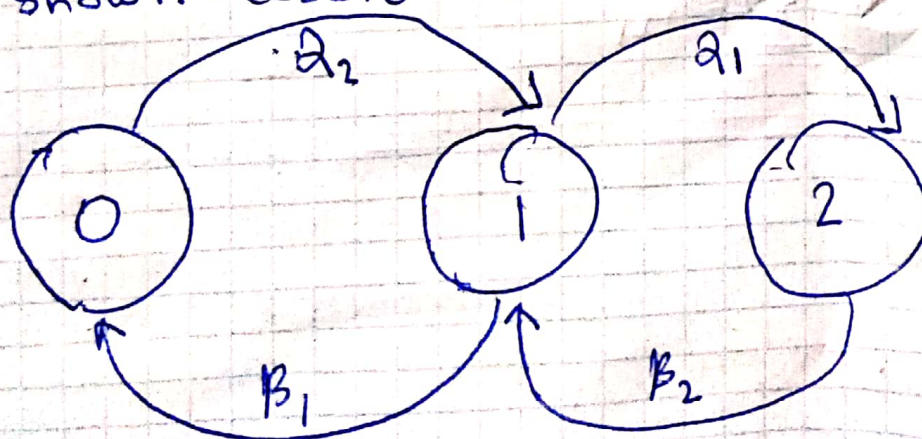


You are getting this from :- 2



⇒ model for birth and death process

Substitute values of λ and μ as shown above to get



Q2 b (ii)

(i) Let $X(t)$ $\rightarrow$ Number of individual in the population at time (t)

$P_n(t)$ $\rightarrow$ The ^{probability} number of the individuals at time (t)

λ $\Rightarrow$ used for number of births

μ $\Rightarrow$ used to denote the number of death.

$$P'_n(t) = P_{n-1}(t)\lambda_{n-1} + P_{n+1}(t)\mu_{n+1} - P_n(t)(\lambda_n + \mu_n)$$

$\nwarrow$
Rate of
Change

$$P'_n(t) = P_{n-1}(t)\lambda_{n-1} + P_{n+1}(t)\mu_{n+1} - P_n(t)(\lambda_n + \mu_n) \quad \text{--- ①}$$

$$P'_0(t) = P_1(t)\mu_1 - P_0(t)\lambda_0 \quad \text{--- ②}$$

① and ② are the equations for ^{modelling} birth and death process

From equation ① equate to zero

$$P_{n-1}(t)\lambda_{n-1} + P_{n+1}(t)\mu_{n+1} - P_n(t)(\lambda_n + \mu_n) = 0$$

From equation ② at $t=0$

$$P_1(t)\mu_1 - P_0(t)\lambda_0 = 0$$

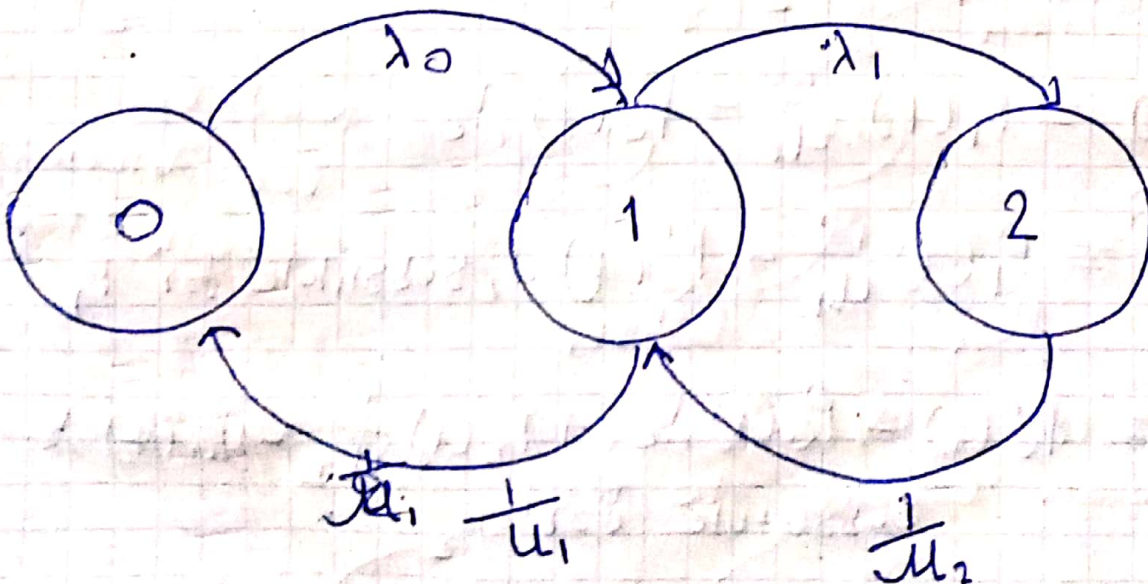
Q2 (c)

(i) $S = \begin{bmatrix} \frac{1}{u} & \frac{u-1}{u} \end{bmatrix} \Rightarrow$ must add up to 1
failure time

$S = \begin{bmatrix} \lambda & 1-\lambda \end{bmatrix} \Rightarrow$ must add up to 1

$$G = \begin{bmatrix} \frac{1}{u} & \lambda \\ \frac{u-1}{u} & 1-\lambda \end{bmatrix} = \begin{bmatrix} \frac{1}{u} & \frac{u-1}{u} \\ \lambda & 1-\lambda \end{bmatrix} \Rightarrow \underline{\underline{\text{Ans}}}$$

Since it can be modeled as a 3-state birth and death process



$$P'_n(t) = P_{n-1}(t)\lambda_{n-1} + P_{n+1}(t)\frac{1}{\mu_{n+1}} - P_n(t)(\lambda_n + \frac{1}{\mu_n}) \quad \text{--- (1)}$$

$$P'_0(t) = P_1(t)\frac{1}{\mu_1} - P_0(t)\lambda_0 \quad \text{--- (11)}$$

$$P_0(t) = P_1(t)\frac{1}{\mu_1} - P_0(t)\lambda_0 \quad \text{--- at } t=0$$

$$P'_1(t) = P_0(t)\lambda_0 + P_2(t)\frac{1}{\mu_2} - P_1(t)(\lambda_1 + \frac{1}{\mu_1})$$

$$P'_2(t) = P_1(t)\lambda_1 + P_3(t)\frac{1}{\mu_3} - P_2(t)(\lambda_2 + \frac{1}{\mu_2})$$

Substitute (P_0, P_1, P_2) from in equation (1) to get above

$$\lambda = \mu = 1, P_0 = \frac{2}{5}$$

$$P'_0(t) = P_1(t)\frac{1}{\mu_1} - P_0(t)\lambda_0$$

$$P_0(t) = P_1(t)\frac{1}{\mu_1} - P'_0(t) \quad , \text{ substitute for } P_1$$

$$P_0(t) = P'_0(t) - P_0(t)\lambda_0 + P_2(t)\frac{1}{\mu_2} - P_0(t)\lambda_0$$

Substitute for P_2

$$P_0(t) = P'_0(t) - P_0(t)\lambda_0 + P_1(t) + P_3(t)\frac{1}{\mu_3} - P'_2(t) - P_0(t)\lambda_0$$

$$= 1 - \frac{3}{5} \quad \text{since } \lambda = \mu = 1$$

$$= \frac{2}{5}$$