

Chapter 12.

2. $\alpha = 0.05$
The probability is greater, because it adds multiple chances of error

4. a) It is because the relative location of the several group means can be more conveniently identified by variance among the groups group means than comparing many group means directly when number of means is large.

b) Because the scores are affected by different factors that distort the values and randomize them.

Treatment 1	Treatment 2	Treatment 3	
$n = 12$	$n = 12$	$n = 12$	$N = 36$
$T = 60$	$T = 72$	$T = 24$	$G = 156$
$SS = 30$	$SS = 46$	$SS = 40$	$\sum x^2 = 896$

~~SS total~~

$$SS_{\text{between}} = \sum \left(\frac{T^2}{n} - \frac{G^2}{N} \right)$$

$$= \sum \left(\frac{60^2}{12} - \frac{156^2}{36} \right) + \left(\frac{72^2}{12} - \frac{156^2}{36} \right) + \left(\frac{24^2}{12} - \frac{156^2}{36} \right) = -1248$$

$$SS_{\text{within}} = \sum x^2 - \frac{\sum T^2}{N}$$

$$= 896 - \left(\frac{60^2}{12} + \frac{72^2}{12} + \frac{24^2}{12} \right) = 636$$

$$SS_{\text{total}} = \sum x^2 - \frac{G^2}{N} = 896 - \frac{156^2}{36} = 220$$

8. 6 treatments = k N = 72

$n = 12$

$df_{\text{between}} = k - 1 = 6 - 1 = 5$

$df_{\text{within}} = N - k = 72 - 6 = 66$

$df_{\text{total}} = N - 1 = 72 - 1 = 71$

10.

$df = 4962$

a) $4 = k - 1$
 $k = 5 = \text{No. of treatments}$

b) $62 = N - 1 \Rightarrow N = 62 + 1 = 63$

c) $F_{\text{critical}} = 2.5201$

d) $F_{\text{critical}} = 3.6378$

12. (a) $Var = \frac{SS}{n-1} = \frac{220}{11} = 20$ for sample 1

For sample 2 $\frac{SS}{n-1} = \frac{60}{6} = 10$

For sample 3 $\frac{SS}{n-1} = \frac{36}{6} = 6$

(b) $MS_{within} \Rightarrow F = \frac{MS_{between}}{MS_{within}}$

$MS = \frac{SS}{df} = \frac{220 + 60 + 36}{2} = 158$

$k = 3$

$df = 2$

$MS = \frac{SS_{within}}{df_{within}}$

$N = 21$

$df_{within} = n - k = 21 - 3 = 18$

$SS_{within} = (n-1)S_1 + (n-1)S_2 + (n-1)S_3$
 $= 11(20) + 6(10) + 6(6) = 316$

$MS_{within} = \frac{316}{18} = 17.55$

14. $n = 5$

Source	SS	df	MS
Between treatments	500	4	125
Within Treatments	1000	20	50
Total	1500	24	175

$F = 2.5$

$N = 5 \times 5 = 25$, $k = 5$

$MS_{between treatments} =$

$df_{between} = k - 1 = 5 - 1 = 4$

$df_{within} = N - k = 25 - 5 = 20$

$MS_{between treatments} = 125 = \frac{SS_{between}}{df_{between}}$

$\Rightarrow SS_{between} = 125 \times df_{between} = 125 \times 4 = 500$

$MS_{within treatments} = \frac{SS_{within}}{df_{within}}$

$SS_{within} = SS_{total} - SS_{between}$
 $= 1500 - 500 = 1000$

$MS_{within} = \frac{1000}{20} = 50$

$F = \frac{MS_{between}}{MS_{within}} = \frac{125}{50} = 2.5$

16. $\alpha = 0.05$
 $n = 8$

for Sample 1

$\bar{x} = 2$

X	$x - \bar{x}$	$(x - \bar{x})^2$
5	3	9
1	-1	1
2	0	0
3	1	1
0	-2	4
1	-1	1
2	0	0
2	0	0

$T = 16$

$\sum = 16 = SS_1$

for Sample 2

$\bar{x} = 3$

X	$x - \bar{x}$	$(x - \bar{x})^2$
2	-1	1
6	3	9
2	-1	1
3	0	0
5	2	4
3	0	0
0	-3	9
3	0	0

$T = 24$

$SS_2 = \sum = 24$

for Sample 3 $\bar{x} = 4$

X	$x - \bar{x}$	$(x - \bar{x})^2$
7	3	9
3	-1	1
2	-2	4
4	0	0
5	1	1
2	-2	4
4	0	0
5	1	1

$T = 32$

$\sum = 20 = SS_3$

For Sample 1 $S_1^2 = \frac{SS_1}{n-1} = \frac{16}{7} = 1\frac{6}{7}$

Sample 2 $S_2^2 = \frac{SS_2}{n-1} = \frac{24}{7} = 3\frac{3}{7}$

Sample 3 $S_3^2 = \frac{SS_3}{n-1} = \frac{20}{7} = 2\frac{6}{7}$

$G = 72$

$N = 24$

$\bar{x}_{Grand} = \frac{72}{24} = 3$

$SS_{within} = (n-1)S_1^2 + (n-1)S_2^2 + (n-1)S_3^2$
 $= 7(1\frac{6}{7}) + 7(3\frac{3}{7}) + 7(2\frac{6}{7}) = 60$

$k = 3 \Rightarrow df_{within} = N - k = 24 - 3 = 21$
 $N = 24$

$\Rightarrow MS_{within} = \frac{SS_{within}}{df_{within}} = \frac{60}{21} = 2\frac{2}{7}$

$MS_{between} = \frac{SS_{between}}{df_{between}}$

$df_{between} = k - 1 = 3 - 1 = 2$

$SS_{between} = \left(\frac{16^2}{8} - \frac{72^2}{24} \right) + \left(\frac{24^2}{8} - \frac{72^2}{24} \right) + \left(\frac{20^2}{8} - \frac{72^2}{24} \right)$
 $= n \sum (\bar{x} - \bar{x}_{Grand})^2$
 $= 8((2-3)^2 + (3-3)^2 + (4-3)^2) = 8 \times 2 = 16$

$MS_{between} = \frac{16}{2} = 8$

$F = \frac{MS_{between}}{MS_{within}} = 8 \times \frac{7}{2} = 28$

$$F_{crit} = 3.4221$$

Calculated value $< F_{crit}$ therefore we cannot reject the null hypothesis
 Therefore no significant difference between the treatments

18. First sample

$$\text{Sum} = 36 \quad \bar{X} = 45/5 = 9 \quad S_1^2 = 1$$

$$\text{Sample 2} \quad \text{sum} = 30 \quad 39/5 = 6 \quad S_2^2 = 4$$

$$\text{Sample 3} \quad \text{sum} = 30 \quad \bar{X} = 30/5 = 6 \quad S_3^2 = 0.5$$

From excel

$$SS \text{ within groups} = 22$$

$$SS \text{ between} = 30$$

$$df \text{ between} = 2 = k - 1 = 3 - 1 = 2$$

$$df \text{ within} = N - k = 15 - 3 = 12$$

$$MS \text{ between} = \frac{SS \text{ between}}{df \text{ between}} = \frac{30}{2} = 15$$

$$MS \text{ within} = \frac{SS \text{ within}}{df \text{ within}} = \frac{22}{12} = \frac{11}{6}$$

$$F = \frac{MS \text{ between}}{MS \text{ within}} = \frac{15 \times 6}{11} = 8.1818$$

$$F(2, 14) = 3.8853$$

Calculated $> F_{critical}$ therefore null hypothesis is rejected
 and we conclude there is significant mean difference

$$20. \quad T = 20, n = 4$$

$$T = 10, n = 5$$

$$T = 30, n = 6$$

$$\bar{X}_1 = T_1/n_1 = 20/4 = 5$$

$$\bar{X}_2 = T_2/n_2 = 10/5 = 2$$

$$\bar{X}_3 = T_3/n_3 = 30/6 = 5$$

$$\bar{X}_{\text{total}} = \frac{G}{N} = \frac{(20+10)}{15} = 4$$

$$SS \text{ between} = n \sum (\bar{X} - \bar{X}_{\text{total}})^2$$

$$= 5((-1)^2 + (-2)^2 + (+1)^2) = 30$$

22. a) I

$$S_1^2 = \frac{SS}{n-1} = \frac{77}{11} = 7$$

$$III = \frac{SS}{n-1} = \frac{66}{11} = 6$$

J

$$S_2^2 = \frac{SS}{n-1} = \frac{110}{11} = 10$$

$$IV = \frac{99}{11} = 9$$

$$\begin{aligned} b) \text{ SS within} &= (n-1)S_1^2 + (n-1)S_2^2 + (n-1)S_3^2 + (n-1)S_4^2 \\ &= 11 \times 7 + 11 \times 10 + 11 \times 6 + 11 \times 9 \\ &= 77 + 110 + 66 + 99 = 352 \end{aligned}$$

$$df_{\text{within}} = N - k = 48 - 4 = 44$$

$$MS_{\text{within}} = \frac{SS_{\text{within}}}{df_{\text{within}}} = \frac{352}{44} = 8$$

23.

- Influences numerator, if difference between means is increased, the SS between increases which directly raises MS between.
- Increasing sample variance increases SS within which increases MS within. Increasing MS reduces the size of F.

24.

$$a) \text{ I } S_1^2 = \frac{SS}{n-1} = \frac{60}{5} = 12$$

$$\text{II } S_2^2 = \frac{SS}{n-1} = \frac{65}{5} = 13$$

$$\text{III } S_3^2 = \frac{SS}{n-1} = \frac{40}{5} = 8$$

$$SS_{\text{between}} = n \sum (\bar{x} - \bar{x}_{\text{Grand}})^2 \quad \bar{x}_{\text{Grand}} = \frac{G}{N} = \frac{54}{18} = 3$$

$$= 6((-2)^2 + (-1)^2 + (3)^2)$$

$$= 6(13) = 78$$

$$df_{\text{within}} = N - k = 18 - 3 = 15$$

$$df_{\text{between}} = k - 1 = 3 - 1 = 2$$

$$\begin{aligned} SS_{\text{within}} &= (n-1)S_1^2 + (n-1)S_2^2 + (n-1)S_3^2 \\ &= 5(12) + 5(13) + 5(8) = 165 \end{aligned}$$

$$MS_{\text{within}} = \frac{SS_{\text{within}}}{df_{\text{within}}} = \frac{165}{15} = 11$$

$$MS_{\text{between}} = \frac{SS_{\text{between}}}{df_{\text{between}}} = \frac{78}{2} = 39$$

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{39}{11} = 3.5454$$

$$F_{\text{crit}} = 3.5915$$

Calculated value < F critical. Therefore null hypothesis cannot be rejected and thus there is