

Chapter 14

2

2.

X	Y
0	4
1	1
0	5
4	1
2	1
1	3

$$M_x = \frac{0+1+0+4+2+1}{6} = 1.333$$

$$M_y = \frac{4+1+5+1+1+3}{6} = 2.5$$

$$\begin{aligned} \sum (x - \bar{x}) \cdot (y - \bar{y}) &= (0 - 1.33) \cdot (4 - 2.5) + (1 - 1.33) \cdot (1 - 2.5) + (0 - 1.33) \cdot (5 - 2.5) + (4 - 1.33) \cdot (1 - 2.5) \\ &\quad + (2 - 1.33) \cdot (1 - 2.5) + (1 - 1.33) \cdot (3 - 2.5) \\ &= (-1.33)(+1.5) + (-0.33)(-1.5) + (-1.33)(2.5) + (2.67)(-1.5) \\ &\quad + (0.67)(-1.5) + (-0.33)(0.5) \\ &= -10 \end{aligned}$$

4. g) Scatter plot.

Gradient is +1

$$\bar{x} = \frac{2+0+6+3}{4} = 3$$

$$\bar{y} = \frac{4+1+6+1}{4} = 3$$

$$\sum (x - \bar{x})^2 = (3-3)^2 + (0-3)^2 + (6-3)^2 + (3-3)^2 = 18$$

$$\sum (y - \bar{y})^2 = (4-3)^2 + (1-3)^2 + (6-3)^2 + (1-3)^2 = 18$$

$$\sum (x - \bar{x})(y - \bar{y}) = (3-3)(4-3) + (0-3)(1-3) + (6-3)(6-3) + (3-3)(1-3) = 15$$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{15}{\sqrt{18 \times 18}} = 0.833$$

6. SS for X

	x^2
11	121
3	9
5	25
6	36
5	25
$\sum x = 30$	$\sum x^2 = 216$

$$\begin{aligned} SS &= \sum x^2 - (\sum x)^2 / n \\ &= 216 - 900 / 5 = 36 \end{aligned}$$

for Y	y^2
1	1
15	225
7	49
8	64
9	81
$\sum y = 40$	$\sum y^2 = 420$

$$\begin{aligned} SS &= \sum y^2 - (\sum y)^2 / n \\ &= 420 - 1600 / 5 = 100 \end{aligned}$$

b

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
5	-7	25	49	-35
-3	7	9	49	-21
-1	-1	1	1	1
0	0	0	0	0
-1	1	1	1	-1
$\Sigma = 0$	0	36	100	-56

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{-56}{\sqrt{36 \times 100}} = -0.9333$$

8. a) Slope of plot = -1.667.

$$b) \bar{x} = \frac{3+5+6+6+5}{5} = 5$$

$$\bar{y} = \frac{6+5+0+2+2}{5} = 3$$

$$\sum (x - \bar{x})^2 = (3-5)^2 + (5-5)^2 + (5-5)^2 + (6-5)^2 + (6-5)^2 + (5-5)^2 = 6$$

$$\sum (y - \bar{y})^2 = (6-3)^2 + (5-3)^2 + (0-3)^2 + (2-3)^2 + (2-3)^2 = 24$$

$x - \bar{x}$	$y - \bar{y}$	$(y - \bar{y})^2$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
-2	3	9	4	-6
0	2	4	0	0
1	-3	9	1	-3
1	-1	1	1	-1
0	-1	1	0	0
$\Sigma = 0$	$\Sigma = 0$	$\Sigma = 24$	$\Sigma = 6$	$\Sigma = -10$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{-10}{\sqrt{(6 \times 24)}} = -0.8333$$

$$10. a) \bar{x} = \frac{1}{6}(4+6+3+9+6+2) = 5$$

$$\bar{y} = \frac{1}{6}(5+5+2+4+5+2) = 4$$

$x - \bar{x}$	$y - \bar{y}$	$(y - \bar{y})^2$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
-1	1	1	1	-1
1	1	1	1	1
-2	-2	4	4	4
4	0	0	16	0
1	1	1	1	1
3	-1	1	9	-3
$\Sigma = 0$	$\Sigma = 0$	$\Sigma = 8$	$\Sigma = 32$	$\Sigma = 2$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$$r = \frac{2}{\sqrt{32 \times 8}} = 0.5$$

b) New pairs

5	5
8	5
5	2
11	4
8	5
5	3

$$\bar{x} = \frac{1}{6} (6 + 8 + 5 + 11 + 8 + 5) = 7.1667$$

$$\bar{y} = \frac{1}{6} (5 + 5 + 2 + 4 + 5 + 2) = 4$$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
-1.1667	1	1.3611	1	-1.1667
0.8333	1	0.6944	1	0.8333
-2.1667	-2	4.6944	4	4.3333
3.8333	0	14.6944	0	0
0.8333	1	0.6944	1	0.8333
-2.1667	-1	4.6944	1	2.1667
$\Sigma = 0$	$\Sigma = 0$	$\Sigma = 26.8333$	$\Sigma = 8$	$\Sigma = 7$

$$r = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2 \Sigma (y_i - \bar{y})^2}}$$

$$= \frac{7}{\sqrt{26.8333 \times 8}} = 0.4776$$

Adding a constant reduces the correlation coefficient therefore it weakens the correlation.

c) New pairs

x	y
8	5
12	5
6	2
18	4
12	5
4	3

$$\bar{x} = \frac{1}{6} (8 + 12 + 6 + 18 + 12 + 4) = 10$$

$$\bar{y} = \frac{1}{6} (5 + 5 + 2 + 4 + 5 + 3) = 4$$

$x - \bar{x}$	$y - \bar{y}$	$(y - \bar{y})^2$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
-2	1	1	4	-2
2	1	1	4	2
-4	-2	4	16	8
8	0	0	64	0
2	1	1	4	2
-6	-1	1	36	6
$\Sigma = 0$	$\Sigma = 0$	$\Sigma = 8$	$\Sigma = 128$	$\Sigma = 16$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{16}{\sqrt{128 \times 6}} = 0.5$$

It does not change the correlation coefficient therefore has no effect on correlation.

12. a) Let cyberbullying score = x and Internet trolling score = y

$$\bar{x} = \frac{1}{7}(-2+4+7+7+6+3+6) = 5$$

$$\bar{y} = \frac{1}{7}(1+8+9+9+5+8) = 7$$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
-3	-6	9	36	18
-1	1	1	1	-1
2	2	4	4	4
2	2	4	4	4
1	2	1	4	2
-2	-2	4	4	-4
1	1	1	1	1
$\Sigma = 0$	$\Sigma = 0$	$\Sigma = 24$	$\Sigma = 54$	$\Sigma = 32$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{32}{\sqrt{24 \times 54}} = 0.8889$$

$$S = \sqrt{\frac{1-r^2}{n-2}} = \sqrt{\frac{1-0.8889^2}{7-2}} = 0.2049$$

$$\text{stat} = \frac{r - 0}{S} = \frac{0.889 - 0}{0.205} = 4.339$$

$$P(X < 4.339) = 0.996 \quad \text{P-value} = 0.00744$$

$$\text{Test statistic} = 4.339$$

$$\text{Critical value} = \pm 2.5706$$

we therefore reject the null hypothesis that difference of sample and expected correlation is big enough to be statistically significant.

$$14. \quad \bar{x} = \frac{1}{10}(1+3+2+3+5+4+5+2+4+2)$$

$$\bar{y} = \frac{1}{10}(3+10+7+11+19+17+17+4+12+3) = 10.3$$

from the table on next page

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{70.7}{\sqrt{16.4 \times 326.1}} = 0.9524$$

Strong positive correlation.

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
-2.1	-7.3	4.41	53.29	15.33
-0.1	-0.3	0.01	0.09	0.02
-1.1	-3.3	1.21	10.89	3.62
-0.1	0.7	0.01	0.49	-0.07
1.9	8.7	3.61	75.69	16.53
0.9	6.7	0.81	44.89	6.03
1.9	6.7	3.61	44.89	12.73
-1.1	-6.3	1.21	39.69	6.93
0.9	1.7	0.81	2.89	1.53
1.1	-7.3	1.21	53.29	8.03
$\Sigma = 0$	$\Sigma = 0$	16.9	$\Sigma = 326.1$	70.7

16.

Rank

X	Y
6	8
7.5	5
4.5	2
3	4
1.5	1
4.5	6
1.5	3
7.5	7

X contains ties
Y does not

$$\bar{x} = \frac{1}{8}(6 + 7.5 + 4.5 + 3 + 1.5 + 4.5 + 1.5 + 7.5) = 4.5$$

$$\bar{y} = \frac{1}{8}(8 + 5 + 2 + 4 + 1 + 6 + 3 + 7) = 4.5$$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
1.5	3.5	2.25	12.25	5.25
3	0.5	9	0.25	1.5
0	-2.5	0	6.25	0
-1.5	-0.5	2.25	0.25	0.75
-3	-3.5	9	12.25	10.5
0	1.5	0	2.25	0
-3	-1.5	9	2.25	4.5
3	2.5	9	6.25	7.5
$\Sigma = 0$	$\Sigma = 0$	$\Sigma = 40.5$	$\Sigma = 42$	30

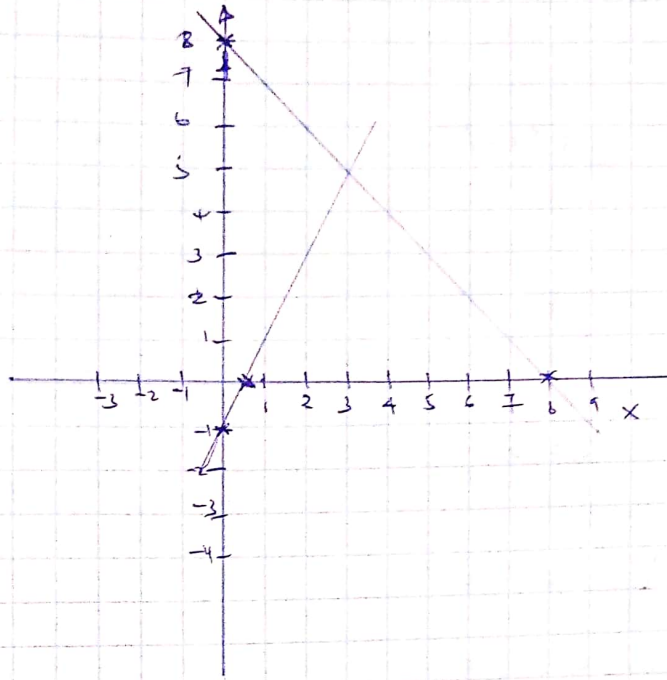
$$r = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2 \Sigma (y_i - \bar{y})^2}} = \frac{30}{\sqrt{40.5 * 42}} = 0.7274$$

18. $y = 2x - 1$

x	0	$\frac{1}{2}$
y	-1	0

$y = -x + 8$

y	0	8
x	8	0



20. $n = 15$ pairs

$y = 3x + 8$

When $x = 1$, $y = 3(1) + 8 = 11$

When $x = 2$, $y = 3(2) + 8 = 14$

When $x = 3$, $y = 3(3) + 8 = 17$

When $x = 6$, $y = 3(6) + 8 = 26$

22. The closer the correlation is to $+1.00$ or -1.00 the smaller the standard error of estimate

24(a) $\bar{x} = \frac{1}{6}(1+2+0+3+2+4) = 2$

$\bar{y} = \frac{1}{6}(0+0+8+14+12+16) = 10$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
-1	-10	1	100	10
0	0	0	0	0
-2	-2	4	4	4
1	4	1	16	4
0	2	0	4	0
2	6	4	36	12
$\Sigma = 0$	$\Sigma = 0$	$\Sigma = 10$	$\Sigma = 160$	$\Sigma = 30$

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$$r = \frac{30}{\sqrt{10 \times 160}} = 0.75$$

$$\sum x = 12 \quad \bar{x} = 12/6 = 2$$

$$\sum y = 60 \quad \bar{y} = 60/6 = 10$$

$$SS_x =$$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})(x - \bar{x})$
-1	-10	1	10
0	0	0	0
-2	-2	4	4
1	4	1	4
0	2	0	0
2	6	4	12
		$\sum = 10$	$\sum = 30$

$$SS_x = 10$$

$$SP = 30$$

$$\hat{y} = bx + a$$

$$b = SP / SS_x = 30/10 = 3$$

$$a = \bar{y} - b\bar{x} = 10 - 3(2) = 4$$

$$\text{Therefore } y = 3x + 4$$

c) b is a character that explains the slope of the line describing the data while the correlation coefficient explains the strength of the correlation.

$$26. \quad \sum x = 21 \quad \bar{x} = 21/7 = 3$$

$$\sum y = 42 \Rightarrow \bar{y} = 42/7 = 6$$

$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(x - \bar{x})(y - \bar{y})$
0	2	0	0
2	2	4	4
-1	0	1	0
-1	-3	1	3
1	0	1	0
-2	-2	4	4
1	1	1	1
		$\sum = 12$	$\sum = 12$

$$\text{Regression eqn } \hat{y} = bx + a$$

$$b = \frac{\sum (x - \bar{x})^2}{\sum (x - \bar{x})(y - \bar{y})} = \frac{12}{12} = 1$$

$$a = \bar{y} - b\bar{x} = 6 - 1(3) = 3$$

$$y = 1x + 3 = x + 3$$

$$y = x + 3$$

b) $y = x + 3$

for :

$$x = 3 \quad y = x + 3 = 3 + 3 = 6$$

$$x = 5 \quad y = 5 + 3 = 8$$

$$x = 2 \quad y = 2 + 3 = 5$$

$$x = 2 \quad y = 2 + 3 = 5$$

$$x = 4 \quad y = 4 + 3 = 7$$

$$x = 1 \quad y = 1 + 3 = 4$$

$$x = 4 \quad y = 1 + 4 = 5$$