

Mini-Lecture 3.1

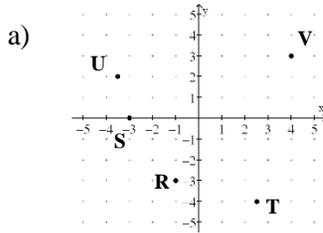
Graphing Equations

Learning Objectives:

1. Plot ordered pairs.
2. Determine whether an ordered pair of numbers is a solution of an equation in two variables.
3. Graph linear equations.
4. Graph nonlinear equations.

Examples:

1. Determine the ordered pairs, or plot the points. Name the quadrant in which each point lies.



b) $(4,2)$; $(-3,5)$; $(-2,-4)$; $(3,-4)$; $(0,5)$; $(-2.5,0)$

2. Determine whether each ordered pair is a solution of the given equation.

a) $x + y = 7$; $(1,6)$, $(-3,10)$ b) $y = -3x + 2$; $(0,2)$, $(-2,10)$ c) $4x - 3y = 1$; $\left(\frac{1}{2}, \frac{2}{3}\right)$, $(0,1)$

3. Graph each linear equation by finding any three ordered pairs that are solutions to the equation.

a) $x + y = 2$ b) $2x - 4y = 8$ c) $y = \frac{2}{3}x + 3$ d) $x = 3$ e) $y = -2$

4. Graph each nonlinear equation by finding any 5 ordered pairs that are solutions to the equation.

a) $y = 3x^2$ b) $y = x^2 - 2$ c) $y = x^3$

Mini-Lecture 3.2

Introduction to Functions

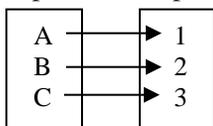
Learning Objectives:

1. Define relation, domain, and range.
2. Identify functions.
3. Use the vertical line test for functions.
4. Find the domain and range of a function.
5. Use function notation.

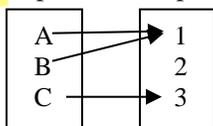
Examples:

1. Find the domain and range of each relation. Also determine whether the relation is a function.

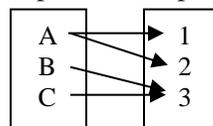
a) Input: Output:



b) Input: Output:



c) Input: Output:



d) $\{(1,4),(1,6)\}$

e) $\{(-2,-6),(0,-6)\}$

f) $\{(-6,-7), (-2,-5), (\frac{1}{2}, \frac{2}{3}), (0.5, 3)\}$

2. Determine whether each relation is also a function.

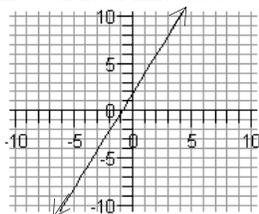
a) $y = x + 3$

b) $y - x = 5$

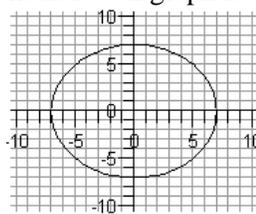
c) $x = 3y^2$

3. Use the vertical line test to determine whether each graph is the graph of a function.

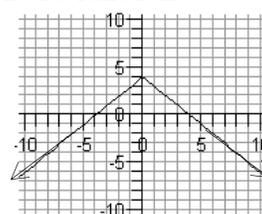
a)



b)



c)



4. Refer to the graphs in problem 3 to answer this question.
Find the domain and range of each relation.

5. For each function, find the indicated values.

a) $f(x) = x - 2$; find $f(3), f(-1)$

b) $g(x) = 3x^2 - 4x + 1$; find $g(0), g(-2)$

Mini-Lecture 3.3

Graphing Linear Functions

Learning Objectives:

1. Graph linear functions.
2. Graph linear functions by using intercepts.
3. Graph vertical and horizontal lines.

Examples:

1. Graph each linear function.

a) $f(x) = x$

b) $f(x) = -2x + 1$

c) $f(x) = 2x - 3$

2. Find the intercepts and graph. Then write each equation using function notation.

a) $4x + 3y = 12$

b) $y = -4x$

c) $x - y = 4$

3. Graph vertical and horizontal lines.

a) $x = -5$

b) $y = -2$

c) $x - 4 = 0$

Mini-Lecture 3.4

The Slope of a Line

Learning Objectives:

1. Find the slope of a line given two points on the line.
2. Find the slope of a line given the equation of the line.
3. Interpret the slope-intercept form in an application.
4. Find the slopes of horizontal and vertical lines.
5. Compare the slopes of parallel and perpendicular lines.

Examples:

1. Find the slope of the line given two points on the line.

a) $(1, 5), (6, 11)$

b) $(3, 6), (-2, 9)$

c) $(3, -1), (4, -5)$

2. Find the slope and the *y-intercept* of each line.

a) $y = x + 3$

b) $y = -4x - 1$

c) $-3x + y = 9$

d) $x = 3.4$

e) $y = -\frac{1}{3}x$

f) $2x - 9y = 36$

g) $y - 8 = 0$

3. Solve.

a) When a road-side service truck is called, the cost of the service is given by the linear function $y = 2x + 60$, where y is in dollars and x is the number of hours the car is worked on. Find and interpret the slope and *y-intercept* of the linear equation.

b) The amount of water in a leaky water jug is given by the linear function $y = 117 - 10x$, where y is in ounces and x is in minutes. Find and interpret the slope and *y-intercept* of the linear function.

4. Find the slope of each line.

a) $x = 3$

b) $x - 5 = 0$

c) $y = -4$

5. Determine whether each pair of lines is parallel, perpendicular, or neither.

a) $y = 3x - 4$

b) $-2x + 4y = 1$

c) $y = 3x + 4$

a) $y = 3x + 2$

b) $6x + 3y = 3$

c) $y = -3x + 4$

Mini-Lecture 3.5

Equations of Lines

Learning Objectives:

1. Graph a line using its slope and y-intercept.
2. Use the slope-intercept form to write the equation of a line.
3. Use the point-slope form to write the equation of a line.
4. Write equations of vertical and horizontal lines.
5. Write equations of parallel and perpendicular lines.

Examples:

1. Graph each linear equation using the slope and y-intercept.

a) $y = 2x$

b) $y = 2x + 3$

c) $y = -2x + 1$

d) $y = \frac{1}{2}x - 2$

e) $x + 2y = 6$

f) $3x - 2y = 12$

2. Use the slope-intercept form of a linear equation to write the equation of each line with the given slope and y-intercept.

a) slope -1 ; y-intercept $(0,4)$

b) slope $\frac{1}{3}$; y-intercept $(0,-7)$

c) slope $-\frac{5}{2}$; y-intercept $(0,0)$

3. Write an equation of each line with the given slope and containing the given point. Write the final equation in slope-intercept form.

a) slope 3 ; through $(6,2)$

b) slope $-\frac{2}{3}$; through $(1,-5)$

c) slope $\frac{3}{2}$; through $(-2,-7)$

Write an equation of the line passing through the given points. Write the final equation in standard form.

d) $(3,0)$ and $(5,4)$

e) $(8,-4)$ and $(5,5)$

f) $\left(-\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{5}{2}, -\frac{2}{3}\right)$

4. Write an equation of each line.

a) vertical; through $(2,4)$

b) horizontal; through $(-1,-3)$

c) undefined slope; through $(0,3)$

d) slope 0 ; through $(-6,4)$

5. Write an equation of each line. Write the equation in the form $x = a$, $y = b$, or $y = mx + b$.

a) through $(0,3)$; parallel to $y = 2x - 1$

b) through $(1,4)$; parallel to $2x - 3y = 1$

c) through $(0,-2)$; perpendicular to $y = -4x + 2$

d) through $(-6,4)$; perpendicular to $2x + 5y = 10$

Mini-Lecture 3.6 (SKIP)

Graphing Piecewise-Defined Functions and Shifting and Reflecting Graphs of Functions

Learning Objectives:

1. Graph piecewise-defined functions.
2. Vertical and horizontal shifts.
3. Reflect graphs.

Examples:

1. Graph each piecewise-defined function.

$$\text{a) } f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x+2 & \text{if } x > 0 \end{cases} \quad \text{b) } g(x) = \begin{cases} 4x+3 & \text{if } x \leq 1 \\ \frac{1}{3}x-2 & \text{if } x > 1 \end{cases}$$

Graph each piecewise-defined function. Use the graph to determine the domain and range.

$$\text{c) } g(x) = \begin{cases} x+2 & \text{if } x < 0 \\ -x+2 & \text{if } x \geq 0 \end{cases} \quad \text{d) } h(x) = \begin{cases} -2 & \text{if } x \leq 0 \\ 2 & \text{if } x \geq 1 \end{cases}$$

2. Sketch each pair of functions on one axis.

$$\begin{array}{lll} \text{a) } f(x) = x & \text{b) } f(x) = |x| & \text{c) } f(x) = |x| \\ g(x) = x+2 & g(x) = |x|-2 & g(x) = |x-2| \\ \\ \text{d) } f(x) = |x| & \text{e) } f(x) = x^2 & \text{f) } f(x) = \sqrt{x} \\ g(x) = |x+2| & g(x) = (x-2)^2 + 1 & g(x) = \sqrt{x+1} - 2 \end{array}$$

3. Sketch each pair of functions on one axis.

$$\begin{array}{ll} \text{a) } f(x) = x & \text{b) } f(x) = |x| \\ g(x) = -x & g(x) = -|x| \\ \\ \text{c) } f(x) = \sqrt{x} & \text{d) } f(x) = x^2 \\ g(x) = -\sqrt{x-2} & g(x) = -(x+2)^2 - 1 \end{array}$$

Teaching Notes:

- Most students find vertical shifts easy to understand.
- Some students are confused by the directions of a horizontal shift.
- Objectives 2 and 3 can be covered in a more timely manner if students are broken into groups and each group is given one type of common graph to focus on. Then the class can discuss the results and generalize to arrive at the shifting and reflecting properties.
- Refer students to the *Vertical Shifts*, *Horizontal Shifts*, and *Reflections About the x-axis* charts in the text.

Answers: (graphing answers at end of mini-lectures) 1c) domain $(-\infty, \infty)$, range $(-\infty, 2]$; d) domain $(-\infty, 0] \cup [1, \infty)$, range $\{-2, 2\}$

Mini-Lecture 3.7

Graphing Linear Inequalities

Learning Objectives:

1. Graph linear inequalities.
2. Graph the intersection or union of two linear inequalities.

Examples:

1. Graph each inequality. Use a test point to check the solution region.

a) $y < x$

b) $y \geq x + 2$

c) $y \leq -x - 3$

d) $x + 2y > -2$

e) $-2x - 5y \geq 10$

f) $2x < -3y$

g) $y > \frac{1}{2}x$

h) $y \leq 2$

i) $x \geq -2\frac{1}{3}$

2. Graph each union or intersection.

a) The intersection of $x \leq 2$ and $y \geq -3$

b) The union of $x \leq 2$ or $y \geq -3$

c) The intersection of $x - y < 2$ and $x + y \geq 3$

d) The union of $2x - 3y < 6$ or $2x + y \geq 3$