

Mini-Lecture 4.1

Solving Systems of Linear Equations in Two Variables

Learning Objectives:

1. Determine whether an ordered pair is a solution of a system of two linear equations.
2. Solve a system by graphing.
3. Solve a system by substitution.
4. Solve a system by elimination.

Examples:

1. Determine whether the given ordered pair is a solution of the system.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$; (3,1) b) $\begin{cases} y = 4 \\ x = -3y \end{cases}$; (-6,4) c) $\begin{cases} 2x + y = 4 \\ -3x = 2y + 8 \end{cases}$; $\left(\frac{1}{2}, 3\right)$

2. Solve each system by graphing.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ b) $\begin{cases} 2x + 4y = 10 \\ 4x + 3y = 10 \end{cases}$ c) $\begin{cases} y = -x + 3 \\ 2x + 2y = -1 \end{cases}$

3. Use the substitution method to solve each system of equations.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ b) $\begin{cases} \frac{1}{4}x + \frac{1}{4}y = 2 \\ x - y = 2 \end{cases}$ c) $\begin{cases} y = -3x + 8 \\ 12x + 4y = 32 \end{cases}$

4. Use the elimination method to solve each system of equations.

a) $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ b) $\begin{cases} x - 6y = -9 \\ 8x - 6y = -30 \end{cases}$ c) $\begin{cases} x - 4y = -8 \\ -6x - 3y = -6 \end{cases}$

d) $\begin{cases} 3x + 6y = 3 \\ 2x + 9y = -8 \end{cases}$ e) $\begin{cases} 6x - 8y = 8 \\ 12x = 16y + 24 \end{cases}$ f) $\begin{cases} -6x - 4y = -2 \\ -12y = -6 + 18x \end{cases}$

Mini-Lecture 4.2 (SKIP)

Solving Systems of Linear Equations in Three Variables

Learning Objectives:

1. Solve a system of three linear equations in three variables.

Examples:

1. Solve each system.

$$\begin{array}{l} x + y + z = 3 \\ \text{a) } x - y + 2z = -1 \\ 4x + y + z = 15 \end{array}$$

$$\begin{array}{l} 5x + 3y + z = 25 \\ \text{b) } 3x - 3y - z = 7 \\ 4x + y + 4z = 14 \end{array}$$

$$\begin{array}{l} x + 4y + 2z = -7 \\ \text{c) } 5y + 4z = -15 \\ z = -5 \end{array}$$

$$\begin{array}{l} x - y + 4z = 3 \\ \text{d) } 5x + z = 0 \\ x + 3y + z = -9 \end{array}$$

$$\begin{array}{l} \frac{2}{3}x - \frac{1}{2}y + 2z = -18 \\ \text{e) } x - \frac{2}{3}y - \frac{1}{2}z = -12 \\ x - \frac{1}{2}y - z = -8 \end{array}$$

Teaching Notes:

- Students need to be extremely neat and organized to succeed with these.
- Most students prefer to use the elimination method repeatedly.
- Some students prefer to use the substitution method to eliminate the first variable whenever it is possible to do so without generating fractions.
- Most students have trouble visualizing these systems. Refer them to the figures of intersecting planes in the text.
- Refer students to the *Solving a System of Three Linear Equations by the Elimination Method* chart in the text.

Answers: 1a) $(4, 1, -2)$; b) $(4, 2, -1)$; c) $(-1, 1, -5)$; d) $(0, -3, 0)$; e) $(-6, 12, -4)$

Mini-Lecture 4.3

Systems of Linear Equations and Problem Solving

Learning Objectives:

1. Solve problems that can be modeled by a system of two linear equations.
2. Solve problems with cost and revenue functions.
3. Solve problems that can be modeled by a system of three linear equations.

Examples:

1. Use a system of two linear equations to solve each problem.
 - a) One number is 5 less than a second number. Twice the second number is 2 less than 5 times the first. Find the two numbers.
 - b) Two trucks leave a city and head in the same direction. After 7 hours the faster truck is 56 miles ahead of the slower truck. The slower truck has traveled 301 miles. Find the speed of the two trucks.
2. Given the cost function $C(x)$ and the revenue function $R(x)$, find the number of units x that must be sold to break even.
 - a) $C(x) = 95x + 1100$
 $R(x) = 105x$
 - b) $C(x) = 0.3x + 1400$
 $R(x) = 1.3x$
 - c) A gift box manufacturing company recently purchased \$1000 worth of new equipment to make gift boxes to sell. The cost of producing a package of gift boxes is \$0.50 and it is sold for \$3.00. Find the number of packages that must be sold for the company to break even.
3. Use a system of three linear equations to solve the problem.
 - c) Find the values of a , b , and c such that the equation $y = ax^2 + bx + c$ has ordered pair solutions $(-3, -37)$, $(-2, -22)$, and $(1, -1)$.
 - d) A store sells tents, sleeping bags, and camp stools. A customer buys a tent, 2 sleeping bags, and 5 camp stools for \$138. The price of a tent is 9 times the price of a camp stool. The cost of a sleeping bag is \$13 more than the cost of a camp stool. Find the cost of each item.

Mini-Lecture 4.4 (SKIP)

Solving Systems of Equations by Matrices

Learning Objectives:

1. Use matrices to solve a system of two equations.
2. Use matrices to solve a system of three equations.

Examples:

1. Use matrices to solve each system of two linear equations.

a)
$$\begin{aligned} 4x + 5y &= -5 \\ 2x + 8y &= 14 \end{aligned}$$

b)
$$\begin{aligned} 6x + y &= 15 \\ 6x + y &= 21 \end{aligned}$$

c)
$$\begin{aligned} 5x + y &= 7 \\ 6x + 2y &= 6 \end{aligned}$$

2. Use matrices to solve each system of three linear equations.

a)
$$\begin{aligned} 4x - y - 4z &= -7 \\ -8x + 5z &= -13 \\ 3y + z &= 16 \end{aligned}$$

b)
$$\begin{aligned} 2x - y + 6z &= 12 \\ 6x + 7y + 5z &= 43 \\ 2x - 5y + z &= -1 \end{aligned}$$

c)
$$\begin{aligned} 6x - 4y + 5z &= -20 \\ -18x + 12y - 15z &= 60 \\ 18x - 12y + 15z &= -60 \end{aligned}$$

Teaching Notes:

- Point out the similarities between solving systems by matrices and solving by the elimination method.
- Remind students that the equations must be written in standard form before writing the corresponding matrix.
- Most students appreciate seeing how matrices can be solved using a calculator, and are amazed at how easy it is to solve a system of three equations with a calculator.
- Refer students to the *Elementary Row Operations* chart in the text.

Answers: 1a) $(-5, 3)$; b) \emptyset ; c) $(2, -3)$; 2a) $(6, 3, 7)$; b) $(4, 2, 1)$; c) $\{(x, y, z) | 6x - 4y + 5z = -20\}$

Mini-Lecture 4.5

Systems of Linear Inequalities

Learning Objectives:

1. Graph a system of linear inequalities.

Examples:

1. Graph the solutions of each system of two linear inequalities.

a) $y \geq 2x - 4$
 $y \leq -x + 1$

b) $y \leq 2x - 1$
 $x + y > -4$

c) $y \leq 2x + 1$
 $y < -3x$

d) $x + 3y > -6$
 $y < -2$

e) $x \geq -2$
 $y \geq 6$

Graph the solutions of each system of three linear inequalities.

f) $x + y \geq 1$
 $x - y \geq 1$
 $x \leq 4$

g) $2x + 3y \geq 6$
 $x - y \leq 3$
 $y \leq 2$

h) $2x + 3y \leq 6$
 $x - y \geq 3$
 $x \geq 1$

Teaching Notes:

- Remind students to use a dashed line for $<$ or $>$ and a solid line for \leq or \geq .
- Encourage students to use different colors for each line.
- Encourage students to check their graphs using a test point from the solution region.
- Refer students to the *Graphing the Solutions of a System of Linear Inequalities* chart in the text.

Answers: (graphing answers at end of mini-lectures)