8·2 : Properties of the normal distribution

Question 1

Mean, μ	standard deviation, σ	Probability
12	3	$P(X<9) = 0.1587$
30	5	$P(X<25) = 0.1587$
5	2·2	$P(X>6) = 0.3247$
245	18	$P(233<X<242) = 0.1813$

Question 2

$\mu = 165$

$\sigma = 20$

$P(X<190)$; $z = \frac{190-165}{20} = 1.25$

$P(z<1.25) = 0.8944$

Question 3

$\mu = 6.2$

$\sigma = 1.08$

$X = 5$

$z = \frac{5-6.2}{1.08} = -1.111$

$P(X<5) = P(z<-1.11) = 0.1333$

$= 13.33\%$

Question 4

$N = 125$

$\mu = 68$

$\sigma = 8.5$

(a) $X = 78$

$$Z = \frac{78 - 68}{8.5} = 1.1765$$

$P(Z < 1.1765) = 0.8803$

$= 88\%$

Percentile rank: 88

(b) $X = 55$

$$Z = \frac{55 - 68}{8.5} = -1.5294$$

$P(Z < -1.5294) = 0.0631$

$= 6$

(c) $X = 89$

$$Z = \frac{89 - 68}{8.5} = 2.4706$$

$P(Z < 2.4706) = 0.9933$

Percentile rank: 99

Question 5

Ty Cobb: 0.420 , 1911
Ted Williams: 0.406 , 1941
George Brett: 0.390 , 1980

Decade	Mean	standard deviation	z-scores
1910_s	0.266	0.0371	T.C : 4.1509
1940_s	0.267	0.0326	T.W : 4.2638
$1970_s - 1980_s$	0.261	0.0317	G.B : 4.0694

$$\text{Ty's} = \frac{0.420 - 0.266}{0.0371} = 4.1509$$

$$\text{Ted's} = \frac{0.406 - 0.267}{0.0326} = 4.2638$$

$$\text{George's} = \frac{0.390 - 0.261}{0.0317} = 4.0694$$

We can rank the three hitters using their z-scores. Ranking their z-scores from the largest to the smallest ; ie best hitter as the one with the highest positive z-score, we will have.

Rank	Hitter
1	Ted Williams
2	Ty Cobb
3	George Brett

Question 6

$\mu = 224$

$\sigma = 0.03$

$P(223.92 < X < 224.08) = P(-2.6667 < Z < 2.6661)$

$Z(224.08) = \frac{224.08 - 224}{0.03} = 2.6667$

~~223.92~~

$Z(223.92) = \frac{223.92 - 224}{0.03} = -2.6667$

$P(223.92 < X < 224.08) = 0.9923$

$1 - 0.9923 = 0.0077$

$= 0.77\%$

Question 7

$\mu = 27\ mg/L$

$\sigma = 14$

$X = 50$

$Z = \frac{50 - 27}{14}$

$= 1.6429$

$P(X > 50) = P(Z > 1.6429) = 0.0502$

$= 0.0502$

Question 8

$\mu = 56.3$

$\sigma = 8.1$

(a) $P(X < 40)$

$$Z = \frac{40 - 56.3}{8.1} = -2.0123$$

$P(z < -2.0123) = 0.0221$

$= 2.21\%$

(b) $P(50 < X < 60)$

$$Z(50) = \frac{50 - 56.3}{8.1} = -0.7778$$

$$Z(60) = \frac{60 - 56.3}{8.1} = 0.4568$$

$P(-0.7778 < z < 0.4568) = 0.4577$

$= 45.77\%$

(c) $P(X > 66)$

$$Z = \frac{66 - 56.3}{8.1} = 1.1975$$

$P(z > 1.1975) = 0.1156$

$= 11.56\%$

Question 9

$\mu = 572.50$

$\sigma = 26.10$

(a) $P(X > 564)$

$$z = \frac{564 - 572.50}{26.10} = -0.3257$$

$P(z > -0.3257) = 0.6277$

$= 62.77\%$

(b) $P(X > 600)$

$$z = \frac{600 - 572.5}{26.1} = 1.0536$$

$P(Z > 1.0536) = 0.146$

$= 14.6\%$

Question 10

$\mu = 157$

$\sigma = 7$

(a) $P(X > 170)$; $z = \frac{170 - 157}{7}$

$= 1.8571$

$P(Z > 1.8571) = 0.0316$

(b) $P = 0.90$

$Z = \pm 1.64$; 2-tailed

$X = z \times \sigma + \mu$

$X = 157 \pm 1.64 \times 7$

$= (145.52, 168.48)$