

1 (a) Winning amount = X
 $X = \$60, \$36, 0$

$$P(X=60) = \frac{1}{6}$$

$$P(X=36) = P(2 \text{ or } 4 \text{ first roll}) \text{ and } P(6 \text{ second roll})$$

$$= \left(\frac{1}{6} + \frac{1}{6}\right) \times \frac{1}{6} = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$$

$$P(X=0) = 1 - (P(X=60) + P(X=36))$$

$$= 1 - \left(\frac{1}{6} + \frac{1}{18}\right)$$

$$= 1 - \frac{2}{9} = \frac{7}{9}$$

Probability Model

X	0	36	60
$P(X)$	$\frac{7}{9}$	$\frac{1}{18}$	$\frac{1}{6}$

(b) Expected amount of winning = $E(X) = \sum x(P(x))$

$$= 60 \cdot \left(\frac{1}{6}\right) + 36 \cdot \left(\frac{1}{18}\right) + 0 \cdot \left(\frac{7}{9}\right)$$

$$= \$12$$

You will be willing to pay not more than \$12

2 (a) Let's say x = number of correct guesses

In this binomial distribution.

$$n = 10, p = 0.5$$

$$X \sim \text{Bin}(n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X=0) = \binom{10}{0} 0.5^0 (1-0.5)^{10-0} = 0.0010$$

$$P(X=1) = \binom{10}{1} 0.5^1 (1-0.5)^{10-1} = 0.0098$$

$$P(X=2) = \binom{10}{2} 0.5^2 (1-0.5)^{10-2} = 0.0439$$

$$P(X=3) = \binom{10}{3} 0.5^3 \times 0.5^7 = 0.1172$$

$$P(X=4) = \binom{10}{4} 0.5^4 \times 0.5^6 = 0.2051$$

$$P(X=5) = \binom{10}{5} 0.5^5 \times 0.5^5 = 0.2461$$

$$P(X=6) = \binom{10}{6} 0.5^6 \times 0.5^4 = 0.2051$$

$$P(X=7) = \binom{10}{7} 0.5^7 \times 0.5^3 = 0.1172$$

$$P(X=8) = \binom{10}{8} 0.5^8 \times 0.5^2 = 0.0439$$

$$P(X=9) = \binom{10}{9} 0.5^9 \times 0.5^1 = 0.0098$$

$$P(X=10) = \binom{10}{10} 0.5^{10} \times 0.5^0 = 0.0010$$

Probability Model

X	P(X)
0	0.0010
1	0.0098
2	0.0439
3	0.1172
4	0.2051
5	0.2461
6	0.2051
7	0.1172
8	0.0439
9	0.0098
10	0.0010

(b) $\geq 80\% = \frac{8}{10}$ (8 correct guesses and above)

$$\begin{aligned}
 P(X \geq 8) &= P(X=8) + P(X=9) + P(X=10) \\
 &= 0.0439 + 0.0098 + 0.0010 \\
 &= 0.0547
 \end{aligned}$$

(c) It would be more difficult to get 80%. Though the probability of getting a correct answer has not changed, the number of items have increased. The outcome will be more difficult to get 80% or more.