

Integration Assignment

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1. $\int x \sin(3x^2 + 1) dx$

Using substitution method with $u = 3x^2 + 1 \Rightarrow \frac{du}{dx} = 6x$

$$dx = \frac{1}{6}x du$$

$$\int \frac{1}{6} x \sin u du = \frac{1}{6} \int \sin u du$$

$$= \frac{-\cos(u)}{6}$$

Undoing substitution

$$= \frac{-\cos(3x^2 + 1)}{6} + C$$

2. $\int 5\sqrt{x^9} dx = \int x^{9/5}$

solve using power rule for integration

$$= \frac{5x^{9/5}}{9} + C$$

3. $\int 3 \sec^2(3x + 7) dx$ let $u = 3x + 7 \Rightarrow \frac{du}{dx} = 3$
 $dx = \frac{1}{3} du$

$$\therefore \int \sec^2(u) du = \tan u + C_1$$

Undoing substitution

$$\tan(3x + 7) + C$$

$$4. \int (6x^4 + 3) dx$$

Integrating each term of the polynomial using power-rule

$$= \int 6x^4 dx + \int 3 dx$$

$$= \boxed{\frac{6}{5}x^5 + 3x + C}$$

$$5. \int 5^{2x} dx : \text{Let } u = 2x \rightarrow \frac{du}{dx} = 2$$

$$dx = \frac{1}{2} du$$

$$= \frac{1}{2} \int 5^u du \Rightarrow \text{Integrate using exponential rule}$$

$$= \frac{5^u}{2 \ln 5} \quad u = 2x$$

$$= \boxed{\frac{5^{2x}}{2 \ln 5} + C}$$

$$6. \int 3e^{4x} (e^{4x} - 7)^4 dx : \text{Let } u = e^{4x} - 7$$

$$\frac{du}{dx} = 4e^{4x} \Rightarrow dx = \frac{e^{-4x}}{4} du$$

$$\frac{3}{4} \int u^4 du \Rightarrow \text{using power rule} = \frac{u^5}{5} \times \frac{3}{4}$$

$$= \frac{3u^5}{20} \Rightarrow \boxed{\frac{3(e^{4x} - 7)^5}{20} + C}$$

$$7. \int \frac{10x-4}{(5x^2-4x-5)^4} dx \quad : \text{ Let } u = 5x^2-4x-5$$

$$\frac{du}{dx} = 10x-4$$

$$\int \frac{du}{(u)^4} \quad \cdot \text{Applying power rule}$$

$$= -\frac{1}{3u^3}$$

Undoing substitution

$$= \boxed{-\frac{1}{3(5x^2-4x-5)^3} + C}$$

$$8. \int 3(7x+1)^4 dx \quad : \text{ Let } u = 7x+1$$

$$\frac{du}{dx} = 7$$

$$= \frac{3}{7} \int u^4 du$$

$$= \frac{3u^5}{5}$$

$$= \boxed{\frac{3(7x+1)^5}{5} + C}$$

$$9. \int \frac{3}{x^6} dx = \int 3x^{-6} dx$$

$$= 3 \int x^{-6} dx$$

$$= 3 \frac{x^{-5}}{-5} = \boxed{-\frac{3}{5x^5}}$$

$$10. \int \frac{e^{4x-1}}{3} = \frac{1}{3} \int e^{4x-1}$$

$$\text{Let } u = 4x-1, \quad \frac{du}{dx} = 4 \Rightarrow dx = \frac{1}{4} du$$

$$= \frac{1}{12} \int e^u du$$

$$= \frac{e^u}{12}$$

$$= \boxed{\frac{e^{4x-1}}{12} + C}$$

$$11. \int \frac{5}{5x+3} dx$$

$$\text{Let } u = 5x+3 \Rightarrow \frac{du}{dx} = 5, dx = \frac{1}{5} du$$

$$\int \frac{1}{u} du = \ln u$$

$$\boxed{\ln(5x+3) + C}$$

$$12. \int \frac{3}{\sqrt{16-x^2}} dx$$

$$\text{Let } u = \frac{x}{4} \quad \frac{du}{dx} = \frac{1}{4}$$

$$dx = 4 du$$

$$\int \frac{3}{\sqrt{16-16u^2}} du = 3 \int \frac{1}{\sqrt{1-u^2}} du$$

standard integral for arcsin

$$= 3 \arcsin u$$

$$= \boxed{3 \arcsin\left(\frac{x}{4}\right) + C}$$

$$13. \int \sqrt{7-3x} dx : \text{Let } u = 7-3x \quad dx = -\frac{1}{3} du$$

$$= -\frac{1}{3} \int \sqrt{u} du$$

$$= -\frac{2}{3} u^{3/2}$$

$$= \boxed{\frac{-2(7-3x)^{3/2}}{3} + C}$$

$$14. \int x \tan(x^2 + 7) dx \quad : \text{ let } u = x^2 + 7$$

$$dx \Rightarrow \frac{1}{2} x du$$

$$\frac{1}{2} \int \tan(u) du = \frac{1}{2} \int \frac{\sin(u)}{\cos u} du$$

Using Quotient rule $u = \cos(u) \quad \frac{du}{du} = -\sin(u)$

$$= - \int \frac{1}{v} dv$$

$$= \ln v = - \ln \cos(u)$$

$$= \frac{- \ln (\cos (x^2 + 7))}{2} + C$$

$$15. \int_1^3 (3x^2 + 4x) dx = \int_1^3 3x^2 dx + \int_1^3 4x dx$$

$$= \left[x^3 \right]_1^3 + \left[2x^2 \right]_1^3$$

$$= [27 - 1] + [18 - 2]$$

$$= 26 + 16$$

$$= \boxed{42}$$