1. Consider a system of T equations given by

$$y_i = X_i\beta + \eta_i i + v_i$$

where $y_i = (y_{i1}, ..., y_{iT})'$ is $T \times 1$, $X_i = (x_{i1}, ..., x_{iT})'$ is $T \times K$, $v_i = (v_{i1}, ..., v_{iT})'$ is $T \times 1$, and i is a $T \times 1$ vector of ones. Assume that

$$\begin{aligned} E[v_i|X_i] &= 0 \\ V[v_i|X_i] &= \Omega. \end{aligned}$$

   (a) Suppose that you are concerned that η_i might be correlated with X_i. Write down an estimator that addresses this problem. Do not use Ω in your estimator.

   Assume that η_i is a potential problem for (b)-(e).

   (b) If we know what Ω is, write down an estimator that is more efficient than the estimator from (a).

   (c) Derive the asymptotic distributions for the SOLS and GLS estimators from (a) and (b).

   (d) Derive the weighting matrix for the efficient GLS estimator when

$$\begin{aligned} v_{it} &= \varepsilon_{it} - \theta\varepsilon_{i(t-1)} \text{ for } t > 1 \\ v_{i1} &= \varepsilon_{i1}. \end{aligned}$$

   and the ε_{it} are white noise with variance equal to σ^2.

   (e) Suppose that we computed the GLS estimator under the (false) assumption that $V[v_i|X_i] = \Omega$. So, in actuality, we have that $V[v_i|X_i] = \Sigma \neq \Omega$. What is the asymptotic distribution of the GLS estimator now? Is it efficient?

2. Consider the model

$$y_{it} = x_{it}'\beta + \eta_i + v_{it}$$

where x_{it} is a $k \times 1$ vector and η_i is unobserved where $i = 1, ..., N$ and $t = 1, ..., T$. Suppose that T is large but N is fixed, so that our asymptotics will be with respect to T. In addition, suppose that for a given i, the v_{it} are stationary and ergodic.

   (a) Write the model as a system of N equations of the form

$$y_t = W_t\delta + v_t.$$

   Define y_t, W_t, δ and v_t in terms of the model's variables and parameters and be explicit about the dimensions and definitions of each of the vectors.

   (b) Write down an estimator of δ and provide a moment condition such that it is consistent.

(c) How would you estimate the standard errors of your estimator?

3. Consider the model:

$$y_{it} = \eta_i + \rho_i \times t + x'_{it}\beta + u_{it}$$

where $i = 1, ..., N$ and $t = 1, ..., T$. Define $X_i \equiv (x_{i1}, ..., x_{iT})'$, $T \equiv (1, ..., T)'$, $y_i \equiv (y_{i1}, ..., y_{iT})$, $u_i \equiv (u_{i1}, ..., u_{iT})$, and i to be a $T \times 1$ vector of ones. Assume that $E[u_i|X_i] = 0$ and $V[u_i|X_i] = \sigma^2\mathbf{I_T}$.

(a) Write the model as a $T \times 1$ system using the definitions given above.

(b) With the existing assumptions, provide two reasons why OLS of y_i onto X_i might be inconsistent.

(c) Provide definitions of a $(T-1)\times T$ and $(T-2)\times(T-1)$ first difference matrices. Call these D_1 and D_2, respectively.

(d) Using D_1 and D_2 to trasnform the system in part (a), provide an estimator that is robust to endogeneity concerns raised in part (b).

(e) Is the estimator in part (d) efficient? If so, why is it? If not, provide an estimator based off of it that is efficient.