Q1. A linear dynamic system with a plant transfer function $G(s)$ is placed in a negative feedback loop incorporating a proportional controller $H(s)$ as shown in Figure 1. The closed-loop system output is required to present a percentage overshoot of 15% or less in response to a unit step reference signal.

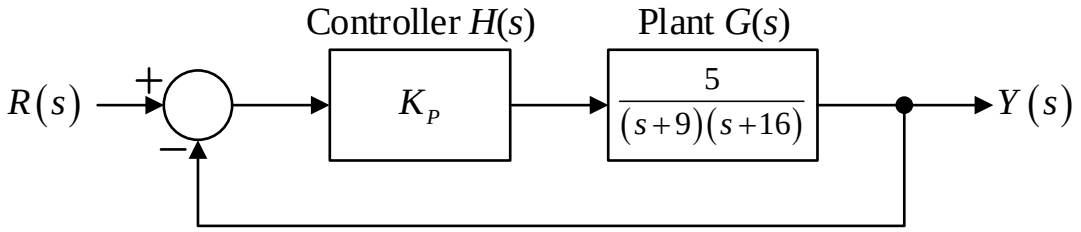


Figure 1

1.a. Briefly explain why this control system will exhibit a steady-state error in response to a unit step reference input. ***[2 marks]***

1.b. Explain how the controller should be modified in order to eliminate the steady-state error when the system is subjected to a unit step reference input. ***[2 marks]***

1.c. Write down the closed-loop transfer function $T(s)=Y(s)/R(s)$. ***[5 marks]***

1.d. Calculate the damping factor ζ and the proportional gain K_P required to achieve the target per-centage overshoot. ***[5 marks]***

1.e. Calculate the natural frequency ω_o, rise time t_r, and 1% settling time t_s that will be achieved for the K_P gain value calculated in 1.d. ***[6 marks]***

Total Marks for Question 1 = 20 marks

Q2. A proportional + integral (PI) feedback controller is applied to a dynamic system (i.e. plant) with the transfer function G(s) given by:

$$G(s) = \frac{1}{(s+1)(s^2+2s+2)}$$

The root locus for this system operating under **proportional feedback** control **only** is shown in Figure 2. The open-loop poles are shown in red and the loci for the closed-loop poles as the proportional gain increases are shown in blue.

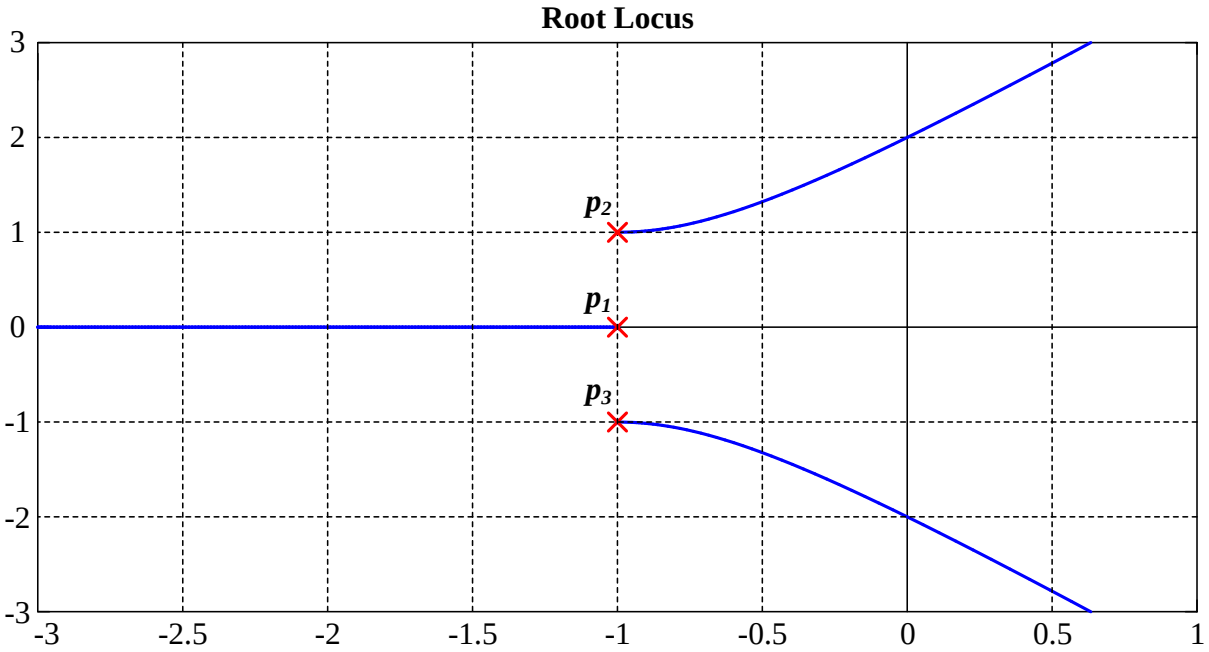


Figure 2

2.a. Sketch the control system feedback block diagram, showing all elements of the PI controller transfer function. ***[5 marks]***

2.b. Using the information provided in Figure 2, determine the critical proportional gain and the associated critical period for which the closed-loop system response exhibits a sustained sinusoidal oscillation. ***[5 marks]***

2.c. Using the result obtained in **2.b** calculate the required PI controller gain constants, and write down a transfer function expression for the resulting PI controller. ***[5 marks]***

2.d. Consider the transfer function $G(s)$ below:

$$G(s) = \frac{K e^{-sT_d}}{Ts+1}$$

where K is the DC gain, T is the time constant, and T_d is the time delay of the system. Can the Ziegler Nichols tuning strategy be applied to determine the parameters of a PI controller for a system with this transfer function G(s)? Justify your answer. ***[5 marks]***

Total Marks for Question 2 = 20 marks

Q3. A first-order plant $G(s)$ is to be controlled using a feedback controller $H(s)$ that incorporates proportional action, as shown in Figure 3.

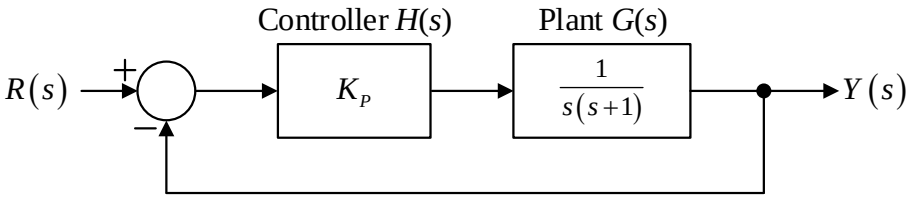


Figure 3

The root locus for this system is shown in Figure 4, where the open-loop poles are shown in red, the loci for the closed-loop poles as the proportional gain increases are shown in blue, and the boundaries defining the transient performance specifications are shown in green.

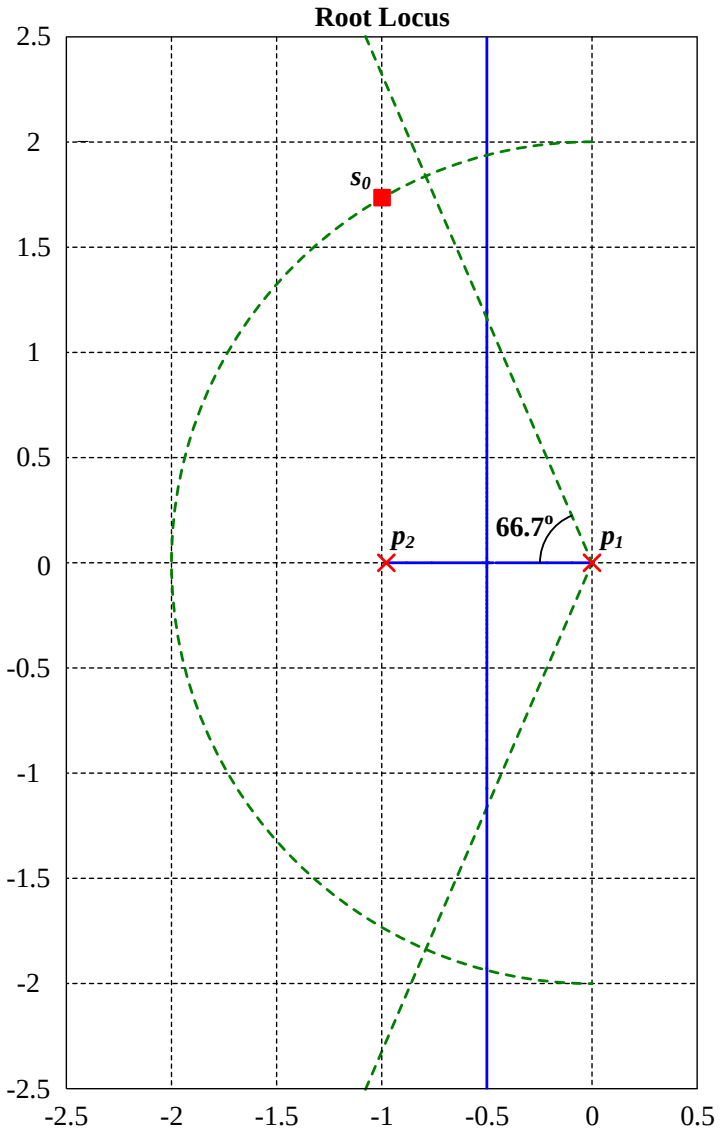


Figure 4

3.a. Using the information provided in Figure 4, identify the natural angular frequency ω_o and the beta angle β associated with the transient performance specifications. Hence determine the required damping ratio ζ, percentage overshoot M_p, damped angular frequency ω_d, and rising time t_r. ***[5 marks]***

3.b. Consider the point $s_o = -1 + j\sqrt{3}$ shown in Figure 4. Calculate the phase of the open-loop plant transfer function $G(s)$ at this location. ***[3 marks]***

3.c. Design a lead compensator to ensure that the root locus passes through the point $s_o = -1 + j\sqrt{3}$ shown in Figure 4. ***[6 marks]***

3.d. Calculate the required proportional gain K_P to move the compensated closed-loop poles p_1 and p_2 to the locations $s = -1 \pm j\sqrt{3}$. ***[6 marks]***

Total Marks for Question 3 = 20 marks

Q4. In the electrical circuit shown in Figure 5 below, the voltages $v_I(t)$ and $v_O(t)$ denote the circuit input and output voltages respectively, with the circuit parameters given by: $R_1 = 10\Omega$, $R_2 = 47\Omega$, $L = 5mH$ and $C = 100\mu F$.

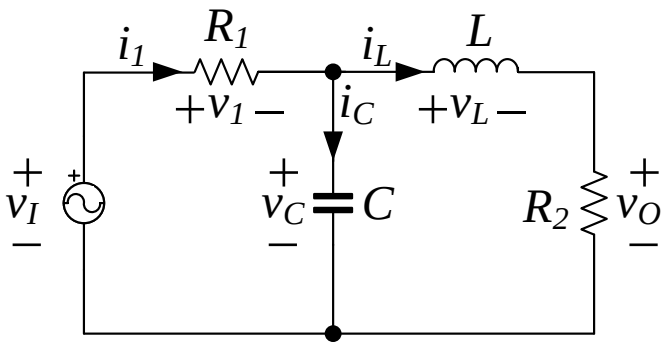


Figure 5

4.a. Identify the dynamic order and appropriate system states for this circuit. ***[4 marks]***

4.b. Write down differential equations for the inductor current and the capacitor voltage respectively. ***[5 marks]***

4.c. Derive the state space equation for this circuit. ***[6 marks]***

4.d. Derive the equivalent transfer function for this circuit, relating the output voltage to the input voltage. ***[5 marks]***

Total Marks for Question 4 = 20 marks

Q5. A linear dynamic system is controlled using a negative feedback proportional control system, as shown in Figure 6 below.

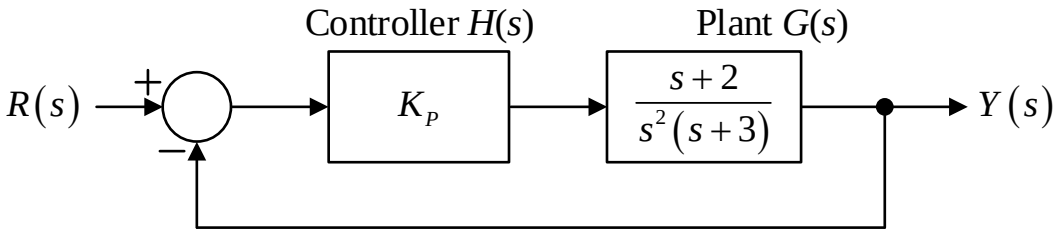


Figure 6

5.a. Sketch the root locus of the closed-loop system poles as a function of the proportional gain constant K_P (on page 9 in this question paper). Your answer must include relevant working, showing in particular:

5.a.i The locations of all open-loop poles and zeros. ***[4 marks]***

5.a.ii All asymptote features, including the centroid location and the angles of each asymptote. ***[4 marks]***

5.a.iii The departure and arrival angles from each open-loop pole and zero. ***[2 marks]***

5.a.iv The location of branch point(s). ***[6 marks]***

5.a.v Provide your final sketch in the Cartesian graph paper provided on page 9 in this question book. ***[4 marks]***

Total Marks for Question 5 = 20 marks

Q6. Consider the feedback control system shown in Figure 7, which incorporates an integral controller $H(s)$ and a plant $G(s)$.

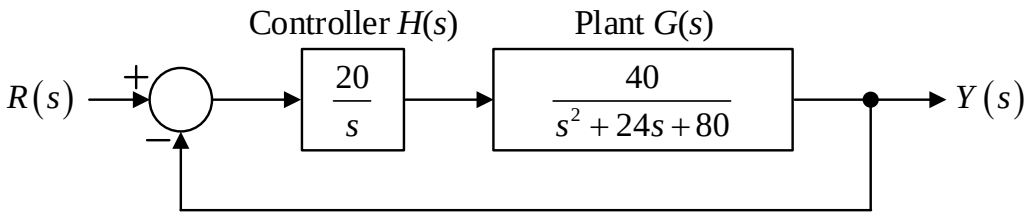


Figure 7

6.a. Identify the control system type, and determine the steady-state error that this system will exhibit in response to a ramp input reference signal (i.e. $r(t) = t,\ t \geq 0$). ***[4 marks]***

6.b. Draw the Bode plot of the forward path gain [i.e. $GH(j\omega)$] for this control system. Use the semi-logarithmic graph paper provided in this question book, and use straight-line approximations for both the magnitude and phase responses. ***[8 marks]***

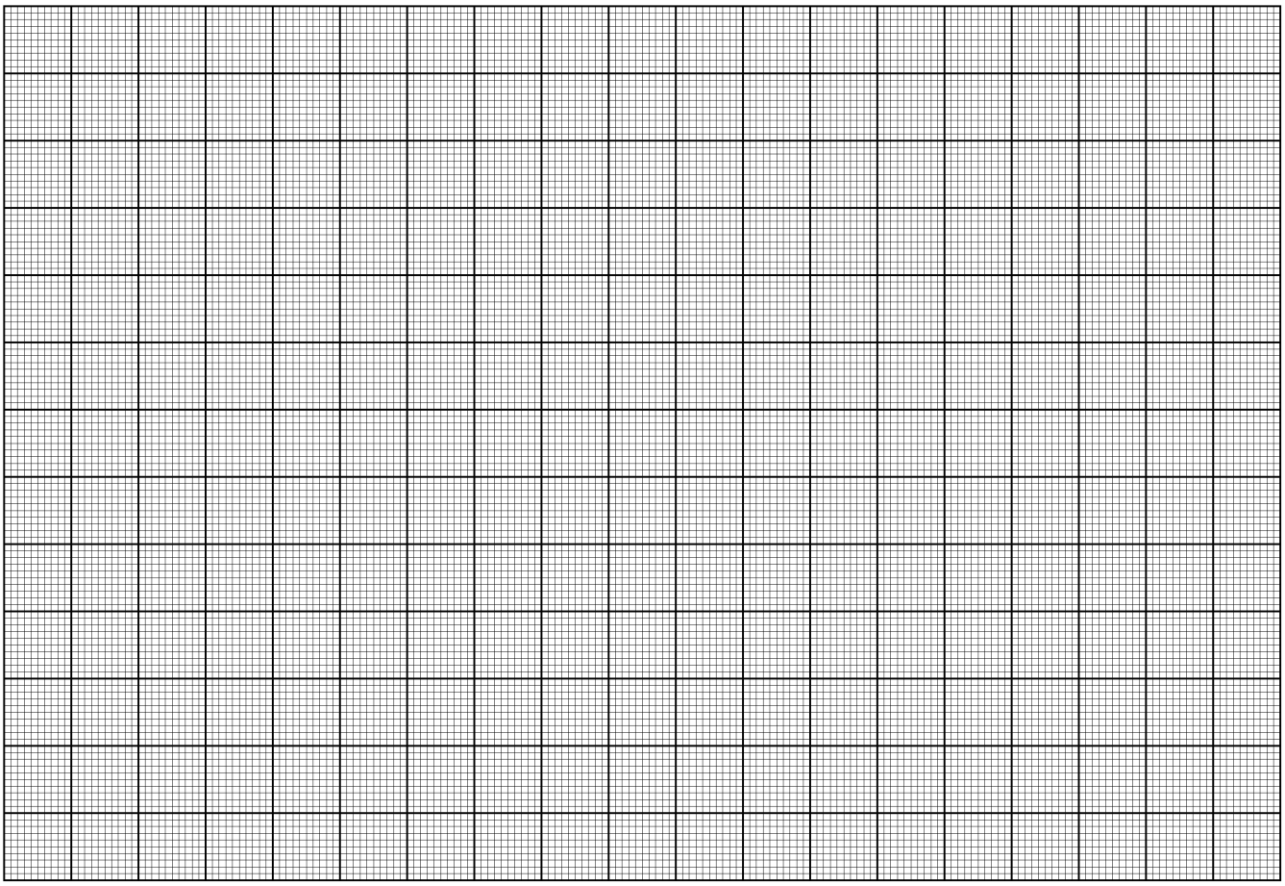
6.c. Using the Bode plot obtained in part **6.b** determine the phase margin and cross-over frequency for this system, and hence identify whether the system is stable. ***[4 marks]***

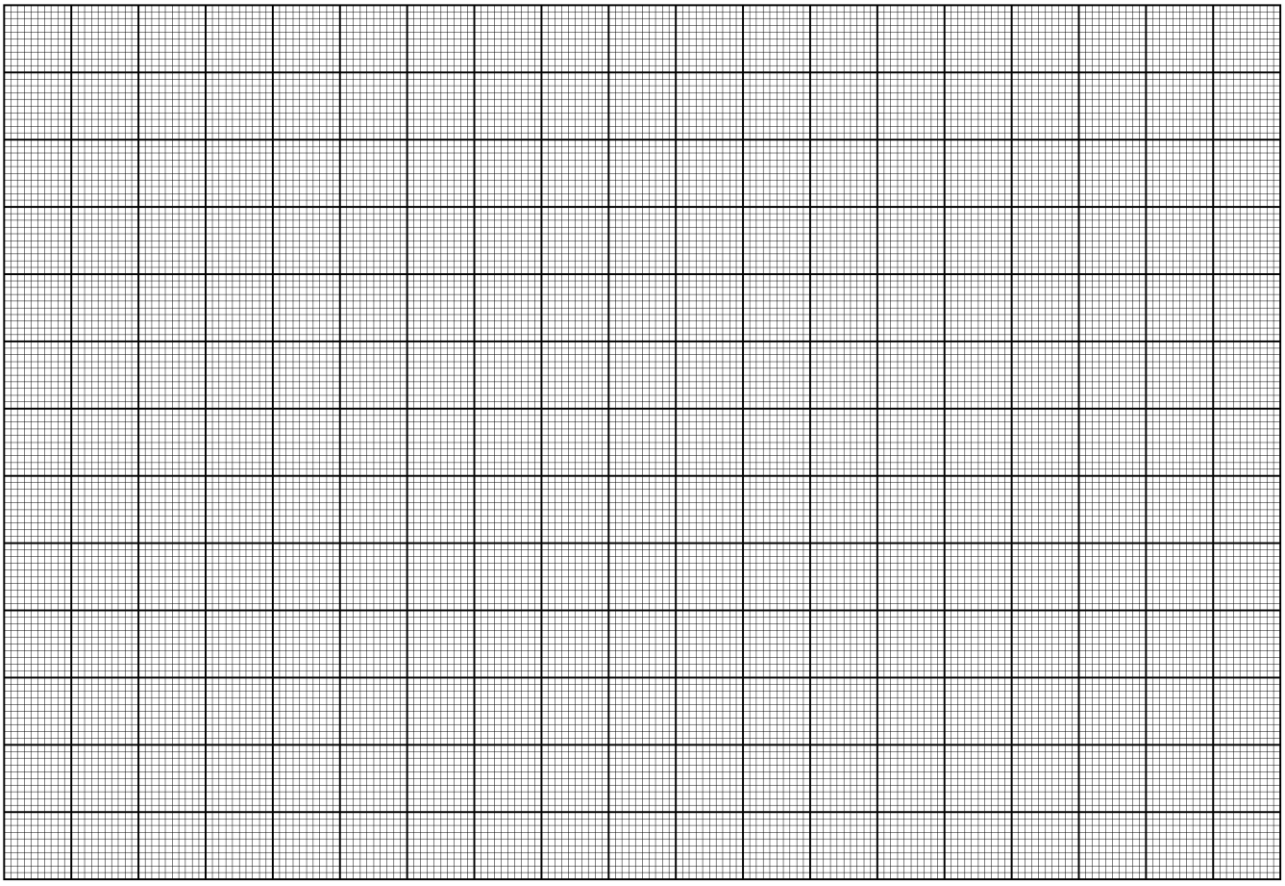
6.d. Design a lead compensator for this control system, so as to achieve a phase margin of at least $\phi_m = 40°$, and a closed-loop system bandwidth of at least $\omega_c = 10$ *rad/s* . ***[4 marks]***

Total Marks for Question 6 = 20 marks

End of Questions

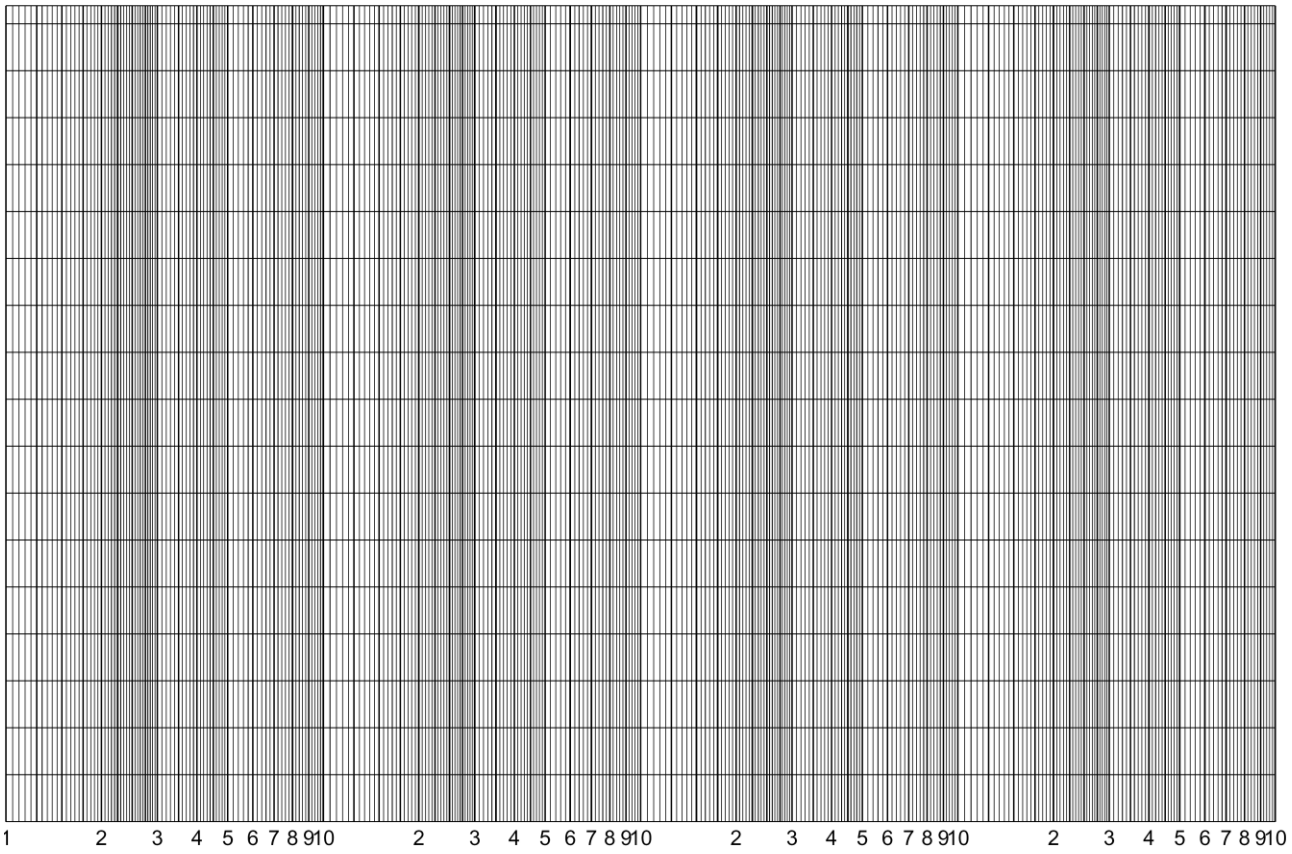
Cartesian Graph Paper

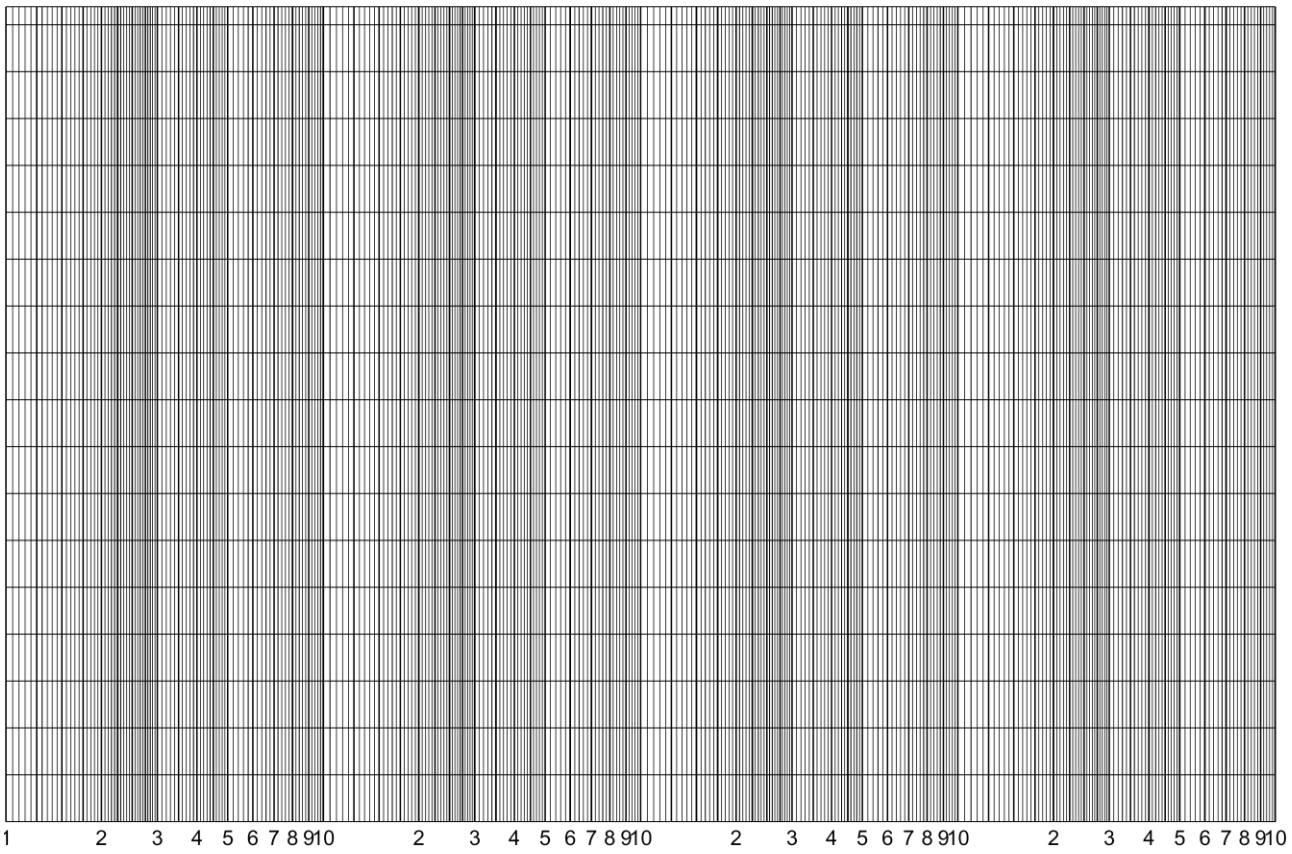




GRAPH PAPER FOLLOWS

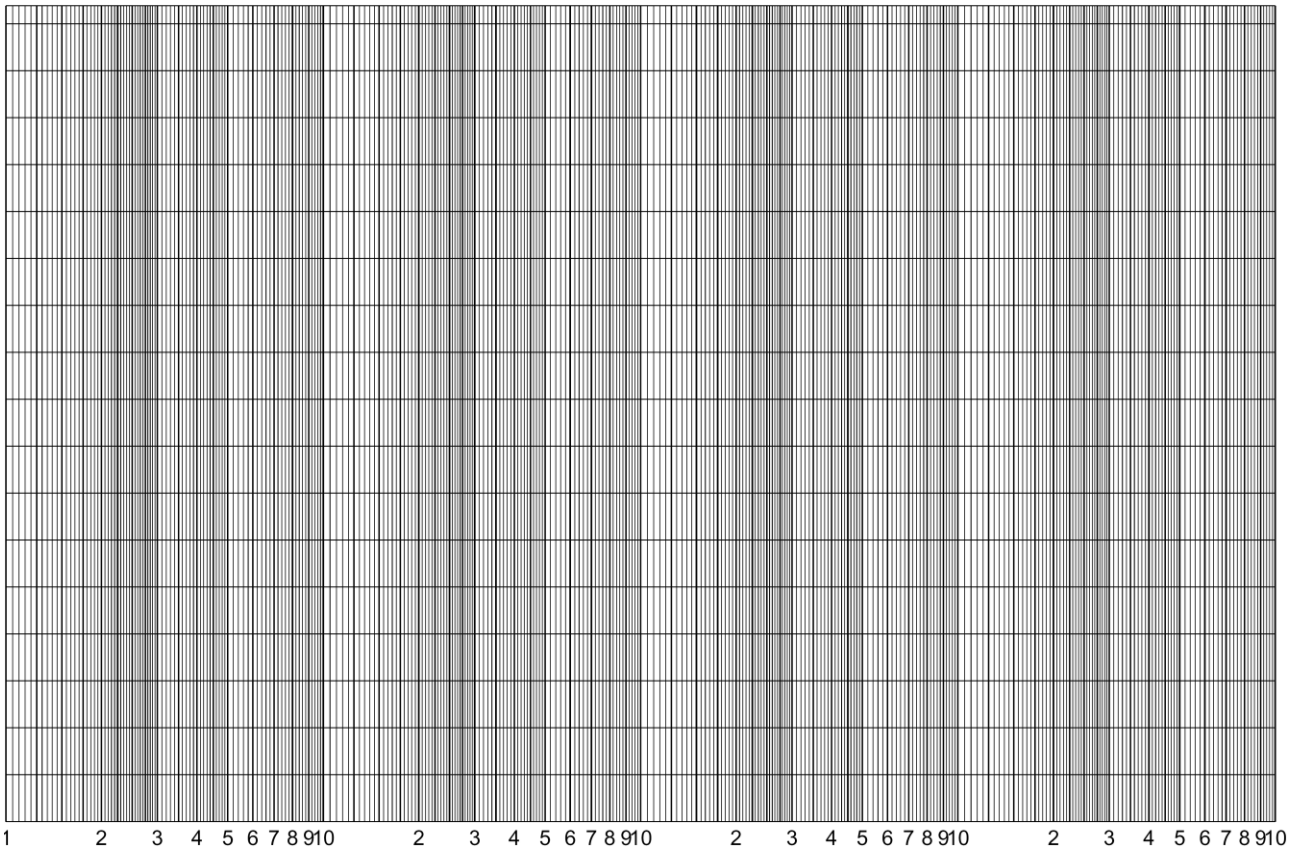
Semi-Logarithmic Graph Paper - 4 Decades

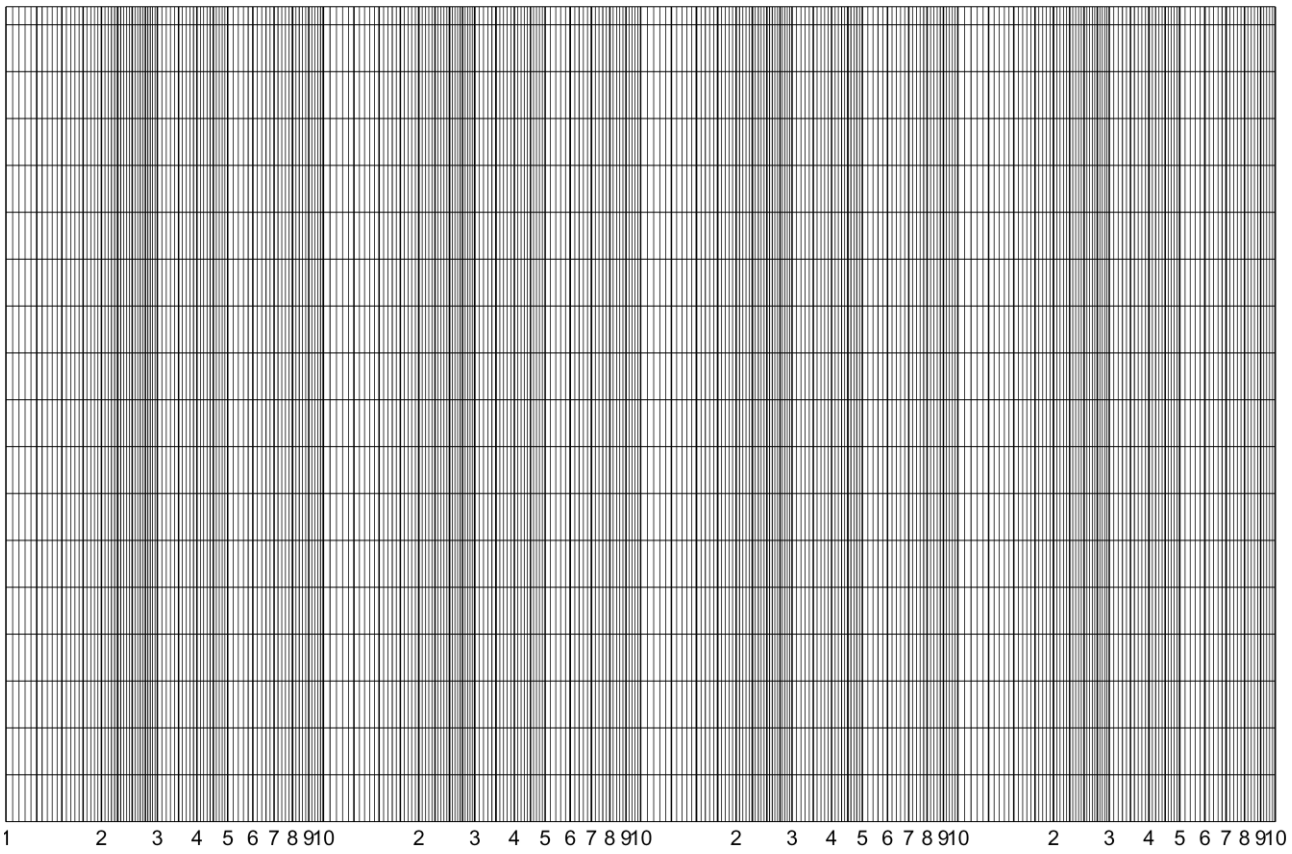




GRAPH PAPER FOLLOWS

Semi-Logarithmic Graph Paper - 4 Decades





FORMULA LIST FOLLOWS

EEET2197 Examination Formula Sheet

Laplace Transforms

$f(t),\ t \geq 0$	$\mathcal{L}\{f(t)\}$	$f(t),\ t \geq 0$	$\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$	$\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} e^{st}F(s)ds$	$\int_0^{\infty} e^{-st}f(t)dt$
$\delta(t)$	1	$\sum_{i=1}^{l} a_i f_i(t)$	$\sum_{i=1}^{l} a_i F_i(s)$
t	$\frac{1}{s^2}$	$\frac{dy(t)}{dt}$	$sY(s)-y(0)$
$t^n,\ \ n \in \{2,3,4,\ldots\}$	$\frac{n!}{s^{n+1}}$	$\frac{d^k y(t)}{dt^k}$	$s^k Y(s)-\sum_{i=1}^{k} s^{k-i}\frac{d^{i-1}y(0)}{dt^{i-1}}$
e^{at}	$\frac{1}{s-a}$	$\int_0^t y(\tau)d\tau$	$\frac{1}{s}Y(s)$
te^{at}	$\frac{1}{(s-a)^2}$	$y(t-\tau)\mu(t-\tau)$	$e^{-s\tau}Y(s)$
$\cos(\omega_o t)$	$\frac{s}{s^2+\omega_o{}^2}$	$ty(t)$	$-\frac{dY(s)}{ds}$
$\sin(\omega_o t)$	$\frac{\omega_o}{s^2+\omega_o{}^2}$	$t^k y(t)$	$(-1)^k\frac{d^k Y(s)}{ds^k}$
$e^{at}\cos(\omega_o t)$	$\frac{s-a}{(s-a)^2+\omega_o{}^2}$	$\int_0^t f_1(\tau)f_2(t-\tau)d\tau$	$F_1(s)F_2(s)$
$e^{at}\sin(\omega_o t)$	$\frac{\omega_o}{(s-a)^2+\omega_o{}^2}$	$\lim_{t\to\infty} y(t)$	$\lim_{s\to 0} sY(s)$
$t\cos(\omega_o t)$	$\frac{s^2-\omega_o{}^2}{\left(s^2+\omega_o{}^2\right)^2}$	$\lim_{t\to 0} y(t)$	$\lim_{s\to\infty} sY(s)$
$t\sin(\omega_o t)$	$\frac{2\omega_o s}{\left(s^2+\omega_o{}^2\right)^2}$	$f_1(t)f_2(t)$	$\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F_1(\xi)F_2(s-\xi)d\xi$
$\mu(t)-\mu(t-\tau)$	$\frac{1-e^{-s\tau}}{s}$	$e^{at}f_1(t)$	$F_1(s-a)$

Transfer Functions

$$G(s)=\frac{a_n s^{n-1}+a_{n-1}s^{n-2}+\cdots+a_2 s+a_1}{s^n+b_n s^{n-1}+b_{n-1}s^{n-2}+\cdots+b_2 s+b_1}$$

$$G(s)=K\frac{\prod_{l=1}^{m}(s+z_l)}{\prod_{k=1}^{n}(s+p_k)} \qquad G(s)=\sum_{k=1}^{n}\frac{c_k}{s+p_k} \qquad c_k=(s+p_k)G(s)\big|_{s=-p_k}$$

$$G(s)=K\frac{\prod_{l=1}^{m}(s+z_l)}{\prod_{k=1}^{r}(s+p_k)^{n_k}} \qquad G(s)=\sum_{k=1}^{r}\sum_{l=1}^{n_i}\frac{c_{ik}}{(s+p_k)^l} \qquad c_{ik}=\frac{1}{(n_k-l)!}\frac{d^{n_{k-1}}}{ds^{n_{k-1}}}\left[(s+p_k)^{n_k}G(s)\right]\Big|_{s=-p_k}$$

Second Order Canonical System

$$G(s)=\frac{{\omega_o}^2}{s^2+2\zeta\omega_o s+{\omega_o}^2} \qquad y(t)=1-\frac{e^{-\zeta\omega_o t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\beta)$$

$$\omega_d=\omega_o\sqrt{1-\zeta^2} \qquad \beta=\sin^{-1}\left(\sqrt{1-\zeta^2}\right) \qquad t_r=\frac{\pi-\beta}{\omega_d} \qquad M_p=e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad t_p=\frac{\pi}{\omega_d}$$

$$\zeta=\sqrt{\frac{\left[\ln(M_p)\right]^2}{\pi^2+\left[\ln(M_p)\right]^2}} \qquad e^{-\zeta\omega_o t_s}=\delta \qquad \zeta\omega_n=\frac{-\ln(\delta)}{t_s}$$

Frequency Response.

$$G(s)=K\frac{\prod_{l=1}^{m}(s/z_l+1)}{s^k\prod_{h=1}^{n}(s/p_h+1)}$$

$$|G(j\omega)|_{dB}=20\log_{10}|K|-20k\log_{10}|\omega|+\sum_{l=1}^{m}20\log_{10}|j\omega/z_l+1|-\sum_{h=1}^{n}20\log_{10}|j\omega/p_h+1|$$

$$\angle G(j\omega)=\angle(K)-k\frac{\pi}{2}+\sum_{l=1}^{m}\angle[j\omega/z_l+1]-\sum_{h=1}^{n}\angle[j\omega/p_h+1]$$

State Space Equations.

$$\begin{aligned}\frac{d\mathbf{x}(t)}{dt} &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)+\mathbf{D}\mathbf{u}(t)\end{aligned} \qquad G(s)=\frac{Y(s)}{U(s)}=\mathbf{C}[s\mathbf{I}-\mathbf{A}]^{-1}\mathbf{B}+\mathbf{D} \qquad \lambda_i,\ \det[\lambda\mathbf{I}-\mathbf{A}]=0 \qquad \mathbf{A}v_i=\lambda_i v_i$$

$$\frac{d\mathbf{x}(t)}{dt}=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\ddots&\vdots\\\vdots&\vdots&\ddots&\ddots&0\\0&0&\cdots&0&1\\-b_1&-b_2&\cdots&-b_{n-1}&-b_n\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}0\\0\\\vdots\\0\\1\end{bmatrix}u(t) \qquad \frac{d\mathbf{x}(t)}{dt}=\begin{bmatrix}0&0&\cdots&0&-b_1\\1&0&\cdots&0&-b_2\\\vdots&1&\ddots&\vdots&\vdots\\0&\ddots&\ddots&0&-b_{n-1}\\0&0&\cdots&1&-b_n\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}a_1\\a_2\\\vdots\\a_{n-1}\\a_n\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}a_1&a_2&\cdots&a_{n-1}&a_n\end{bmatrix}\mathbf{x}(t)+[0]u(t) \qquad y(t)=\begin{bmatrix}0&0&\cdots&0&1\end{bmatrix}\mathbf{x}(t)+[0]u(t)$$

$$\frac{d\mathbf{x}(t)}{dt}=\begin{bmatrix}-\lambda_1&0&\cdots&\cdots&0\\0&-\lambda_2&\ddots&\ddots&\vdots\\\vdots&\ddots&\ddots&\ddots&\vdots\\\vdots&\ddots&\ddots&-\lambda_{n-1}&0\\0&\cdots&\cdots&0&-\lambda_n\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}a_1\\a_2\\\vdots\\a_{n-1}\\a_{n-2}\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1&1&\cdots&1&1\end{bmatrix}\mathbf{x}(t)+[0]u(t)$$

$$\begin{aligned}\frac{d\mathbf{x}(t)}{dt} &= \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t),t)\\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t),\mathbf{u}(t),t)\end{aligned} \qquad \begin{aligned}\frac{d\mathbf{x}(t)}{dt} &= \mathbf{A}\Delta\mathbf{x}(t)+\mathbf{B}\Delta\mathbf{u}(t)+\mathbf{E}\\ \mathbf{y}(t) &= \mathbf{C}\Delta\mathbf{x}(t)+\mathbf{D}\Delta\mathbf{u}(t)+\mathbf{F}\end{aligned}$$

$$\mathbf{A}=\left.\frac{d\mathbf{f}}{d\mathbf{x}}\right|_{\substack{\mathbf{x}=\mathbf{x}_Q\\\mathbf{u}=\mathbf{u}_Q}} \qquad \mathbf{B}=\left.\frac{d\mathbf{f}}{d\mathbf{u}}\right|_{\substack{\mathbf{x}=\mathbf{x}_Q\\\mathbf{u}=\mathbf{u}_Q}} \qquad \mathbf{C}=\left.\frac{d\mathbf{g}}{d\mathbf{x}}\right|_{\substack{\mathbf{x}=\mathbf{x}_Q\\\mathbf{u}=\mathbf{u}_Q}} \qquad \mathbf{D}=\left.\frac{d\mathbf{g}}{d\mathbf{u}}\right|_{\substack{\mathbf{x}=\mathbf{x}_Q\\\mathbf{u}=\mathbf{u}_Q}} \qquad \mathbf{E}=\mathbf{f}(\mathbf{x}_Q,\mathbf{u}_Q) \qquad \mathbf{F}=\mathbf{g}(\mathbf{x}_Q,\mathbf{u}_Q)$$

$$\Delta\mathbf{x}(t)=\mathbf{x}(t)-\mathbf{x}_Q, \qquad \Delta\mathbf{u}(t)=\mathbf{u}(t)-\mathbf{u}_Q$$

PID Controllers.

	K_P	T_r	T_D
P	$0.5K_C$		
PI	$0.45K_C$	$\frac{P_C}{1.2}$	
PID	$0.6K_C$	$\frac{P_C}{2}$	$\frac{P_C}{8}$

$$H_P(s)=K_P \qquad H_{PI}(s)=K_P+\frac{K_I}{s} \qquad H_{PI}(s)=K_P\left(1+\frac{1}{T_r s}\right) \qquad T_r=K_P/K_I$$

$$H_{PD}(s)=K_P\left(1+\frac{T_d s}{\tau_D s+1}\right) \qquad H_{PD}(s)=K_P+K_d s \qquad T_d=K_D/K_P$$

$$H_{PID}(s)=K_P\left(1+\frac{1}{T_r s}+\frac{T_d s}{\tau_D s+1}\right) \qquad H_{PID}(s)=K_P+\frac{K_I}{s}+K_d s$$

Root Locus.

$$G(s)=\frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{k=1}^{n}(s+p_k)} \qquad \angle G(s)=180°+360°(l) \qquad \angle G(s)=\sum_{i=1}^{m}\psi_i-\sum_{k=1}^{n}\phi_k$$

$$\psi_i=\tan^{-1}\left(\frac{\mathrm{Im}(s+z_i)}{\mathrm{Re}(s+z_i)}\right) \qquad \phi_k=\tan^{-1}\left(\frac{\mathrm{Im}(s+p_k)}{\mathrm{Re}(s+p_k)}\right) \qquad b(s)\frac{da(s)}{ds}-a(s)\frac{db(s)}{ds}=0$$

$$\phi=\frac{180°+360°(l-1)}{n-m}, \quad l=1,2,\ldots,n-m \qquad \alpha=\frac{\sum p-\sum z}{n-m}$$

$$q\phi_{dep}=\sum\psi-\sum\phi-180°-360°l \qquad q\psi_{arr}=\sum\phi-\sum\psi+180°+360°l$$

$$K_P=\left|\frac{1}{G(s)}\right|=\frac{|s_o-p_n||s_o-p_{n-1}|\cdots|s_o-p_2||s_o-p_1|}{|s_o-z_m||s_o-z_{m-1}|\cdots|s_o-z_2||s_o-z_1|} \qquad |s_o-p_n|=\sqrt{\mathrm{Re}(s_o-p_n)^2+\mathrm{Im}(s_o-p_n)^2}$$

Dynamic Compensation.

$$D_{lag}(s)=\frac{s+z}{s+p},\ p<z \qquad D_{lead}(s)=\frac{s+z}{s+p},\ z<p \qquad \angle G(s_o)+\angle D_{lead}(s_o)=180°$$

$$E_k=\lim_{s\to 0}\left[s^k K_p D_{lag}(s)G(s)\right]=K_p\frac{z}{p}\lim_{s\to 0}\left[s^k G(s)\right]$$

Nyquist and Bode Design.

$$Z=N+P \qquad \phi_m\approx 100\zeta$$

$$D_{lag}(s)=\frac{s/z+1}{s/p+1}, \quad z>p, \quad 20\log_{10}\left(\frac{p}{z}\right)=D, \quad z\ll\omega_c$$

$$D_{lead}(s)=\frac{s/z+1}{s/p+1}, \quad z<p, \quad \theta_{lead}=90°-2\tan^{-1}\left[\sqrt{\frac{z}{p}}\right], \quad \omega_c=\sqrt{zp}$$

$$20\log 10\left|H_{lag}(j\omega_{sc})H_{lead}(j\omega_{sc})G(j\omega_{sc})\right|=0dB$$

Table of Laplace and Z Transforms.

Laplace Transform	**Impulse Response**	**Z-Transform (Impulse Invariant)**
$H(s)=\mathcal{L}\{h(t)\}$	$h(t)=\mathcal{L}^{-1}\{H(s)\}$	$H(z)=Z\{H(s)\}$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$
$\frac{1}{s^4}$	$\frac{t^3}{6}$	$\frac{T^3 z(z^2+4z+1)}{6(z-1)^4}$
$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
$\frac{a}{s(s+a)}$	$1-e^{-at}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-e^{-at}}{a}$	$\frac{z[(aT-1+e^{-aT})z+(1-e^{-aT}-aTe^{-aT})]}{a(z-1)^2(z-e^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)e^{-at}$	$\frac{z}{z-1}-\frac{z}{z-e^{-aT}}-\frac{aTe^{-aT}z}{(z-e^{-aT})^2}$
$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
$\frac{ab}{s(s+a)(s+b)}$	$1+\frac{be^{-at}}{a(a-b)}+\frac{ae^{-bt}}{b(b-a)}$	$\frac{z}{z-1}+\frac{bz}{(a-b)(z-e^{-aT})}-\frac{az}{(a-b)(z-e^{-bT})}$
$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$	$\frac{z[(b-a)z-(be^{-aT}-ae^{-bT})]}{(z-e^{-aT})(z-e^{-bT})}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos(aT)+1}$

Z Transforms of H(s).

$|H(s=0)|=|H(z=1)|$ for $z=e^{sT}$, (PZ Matching)

$s=\frac{2}{T}\frac{z-1}{z+1}$ (Tustin) $\qquad H(z)=T\sum \text{Residue}\left\{\frac{H(s)}{1-z^{-1}e^{sT}}\right\}$ (Impulse Invariant)

$H(z)=\left[1-z^{-1}\right]Z\left\{\frac{H(s)}{s}\right\}$ (ZOH)

S-Plane to Z-Plane Mapping.

$$z = e^{sT} = e^{(\sigma + j\omega)T} \qquad z = a + jb = \sqrt{a^2 + b^2}\, e^{j\tan^{-1}\left(\frac{b}{a}\right)} \qquad e^{\sigma T} = \sqrt{a^2 + b^2},\ e^{j\omega T} = e^{j\tan^{-1}\left(\frac{b}{a}\right)}$$

$$s = \frac{\ln\sqrt{a^2 + b^2}}{T} \qquad j\omega = j\frac{1}{T}\tan^{-1}\left(\frac{b}{a}\right) \qquad \omega_n = \sqrt{\sigma^2 + \omega^2} \qquad \zeta = \frac{\sigma}{\omega_n}$$

$$\zeta = 1, \text{ for a real pole in LHS} \qquad \zeta = -1, \text{ for a real pole in RHS} \qquad \omega_d = \omega_n\sqrt{1 - \zeta^2}, \text{ for } 0 \le \zeta \le 1$$

Control Canonical Form.

$$G(z) = \frac{b_o z^n + b_1 z^{n-1} + \cdots + b_{m-1} z^{m-1} + b_m z^{n-m}}{z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n},\ n \ge m$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \qquad A = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_{nh} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} (b_1 - a_1 b_0) & (b_2 - a_2 b_0) & \cdots & (b_m - a_m b_0) & -a_n b_0 \end{bmatrix} \qquad D = \begin{bmatrix} b_0 \end{bmatrix}$$

END OF PAPER