

Engineering Economy

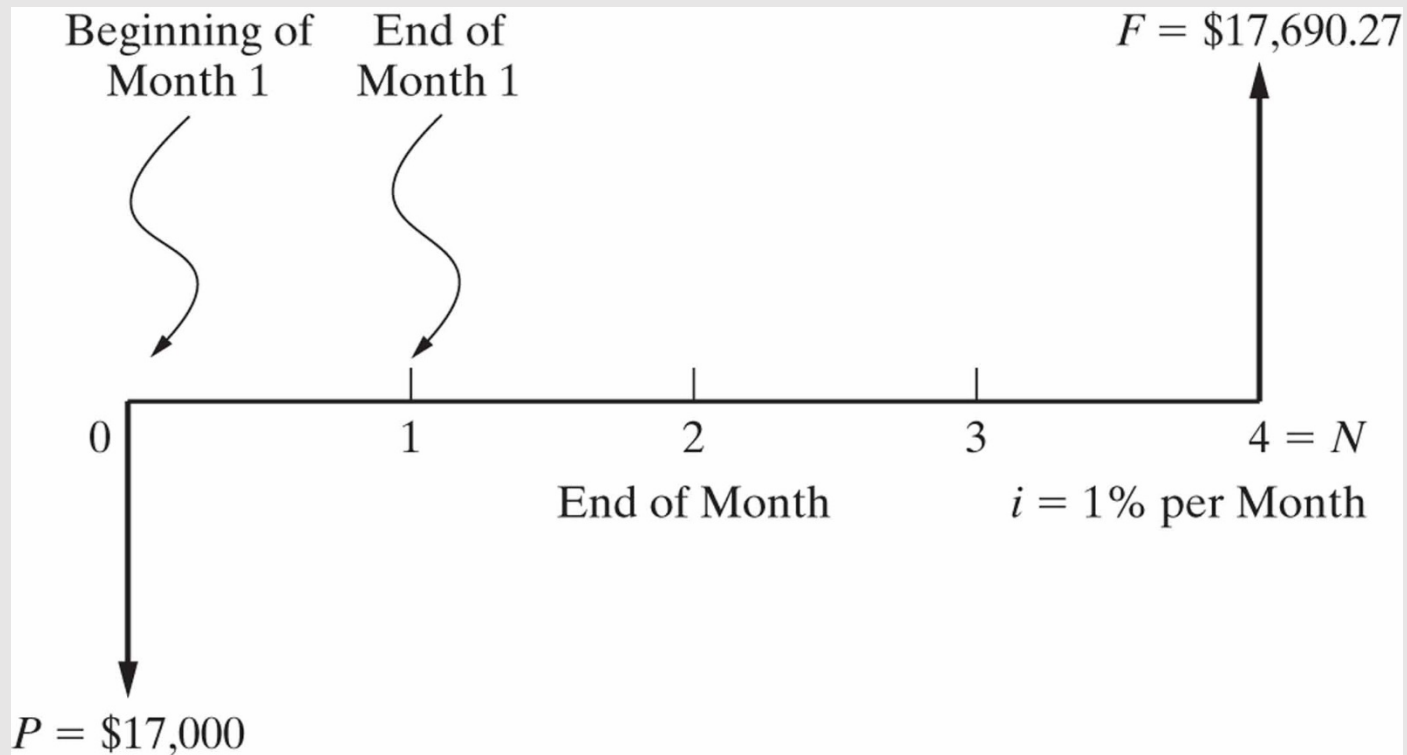
Chapter 4: Equivalence

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We need some tools to find economic equivalence.

- Notation used in formulas for compound interest calculations.
 - i = effective interest rate per interest period
 - N = number of compounding (interest) periods
 - P = present sum of money; *equivalent* value of one or more cash flows at a reference point in time; the present
 - F = future sum of money; *equivalent* value of one or more cash flows at a reference point in time; the future
 - A = end-of-period cash flows in a uniform series continuing for a certain number of periods, starting at the end of the first period and continuing through the last

A cash flow diagram is an great tool for clarifying and visualizing a series of cash flows.



Compounded monthly $\$17,000 (1.01)^4$

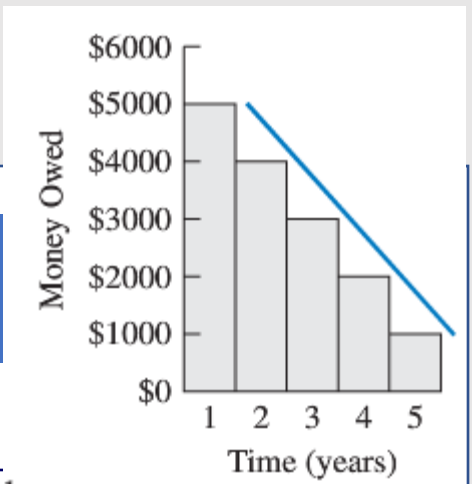
Cash flow tables are good for modeling engineering economy problems in a spreadsheet

	A	B	C	D	E
1		Alternative A	Alternative B	Difference	Cumulative
2	End of Year	Net Cash Flow	Net Cash Flow	(B-A)	Difference
3	0 (now)	\$ (18,000)	\$ (60,000)	\$ (42,000)	\$ (42,000)
4	1	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (32,600)
5	2	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (23,200)
6	3	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (13,800)
7	4	\$ (34,400)	\$ (34,400)	\$ -	\$ (13,800)
8	5	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ (4,400)
9	6	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ 5,000
10	7	\$ (34,400)	\$ (25,000)	\$ 9,400	\$ 14,400
11	8	\$ (32,400)	\$ (17,000)	\$ 15,400	\$ 29,800
12	Total	\$ (291,200)	\$ (261,400)		

$= -25000 - 9400$
 $= C3 - B3$
 $= \text{SUM}(D\$3:D3)$
 $= -34400 + 2000$
 $= \text{SUM}(B3:B11)$
 $= -25000 + 8000$

Plan #1: Constant Principal

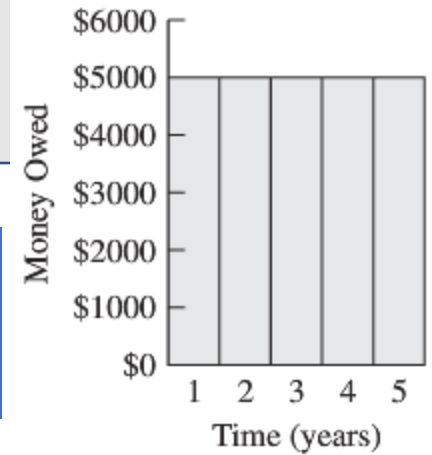
- Repay of a loan of \$5000 in 5 yrs at interest rate of 8%
- Plan #1: Constant principal payment plus interest due



Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$1,000.00	\$1,400.00
2	\$4,000.00	\$320.00	\$4,320.00	\$320.00	\$1,000.00	\$1,320.00
3	\$3,000.00	\$240.00	\$3,240.00	\$240.00	\$1,000.00	\$1,240.00
4	\$2,000.00	\$160.00	\$2,160.00	\$160.00	\$1,000.00	\$1,160.00
5	\$1,000.00	\$80.00	\$1,080.00	\$80.00	\$1,000.00	\$1,080.00
	Subtotal			\$1,200.00	\$5,000.00	\$6,200.00

Plan #2: Interest Only

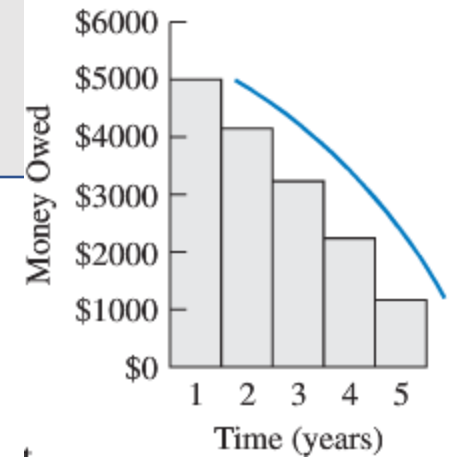
- Repay of a loan of \$5000 in 5 yrs at interest rate of 8%
- Plan #2: Annual interest payment and principal payment at end of 5 yrs



Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
2	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
3	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
4	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$0.00	\$400.00
5	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$5,000.00	\$5,400.00
	Subtotal			\$2,000.00	\$5,000.00	\$7,000.00

Plan #3: Constant Payment

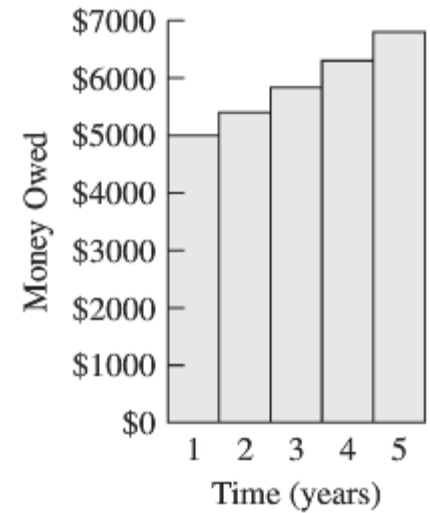
- Repay of a loan of \$5000 in 5 yrs at interest rate of 8%
- Plan #3: Constant annual payments



Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$400.00	\$852.28	\$1,252.28
2	\$4,147.72	\$331.82	\$4,479.54	\$331.82	\$920.46	\$1,252.28
3	\$3,227.25	\$258.18	\$3,485.43	\$258.18	\$994.10	\$1,252.28
4	\$2,233.15	\$178.65	\$2,411.80	\$178.65	\$1,073.63	\$1,252.28
5	\$1,159.52	\$92.76	\$1,252.28	\$92.76	\$1,159.52	\$1,252.28
	Subtotal			\$1,261.41	\$5,000.00	\$6,261.41

Plan #4: All at Maturity

- Repay of a loan of \$5000 in 5 yrs at interest rate of 8%
- Plan #4: All payment at end of 5 years



Yr	Balance at the Beginning of year	Interest	Balance at the end of year	Interest Payment	Principal Payment	Total Payment
1	\$5,000.00	\$400.00	\$5,400.00	\$0.00	\$0.00	\$0.00
2	\$5,400.00	\$432.00	\$5,832.00	\$0.00	\$0.00	\$0.00
3	\$5,832.00	\$466.56	\$6,298.56	\$0.00	\$0.00	\$0.00
4	\$6,298.56	\$503.88	\$6,802.44	\$0.00	\$0.00	\$0.00
5	\$6,802.44	\$544.20	\$7,346.64	\$2,346.64	\$5,000.00	\$7,346.64
	Subtotal			\$2,346.64	\$5,000.00	\$7,346.64

A Closer Look at the 4 Repayment

- Differences:
 - Repayment structure (repayment amounts at various points in time)
 - Total payment amount
- Similarities:
 - All interest charges were calculated at 8%
 - They all achieved the same purpose of repaying the loan within 5 years

Equivalence

- If a firm believes 8% was reasonable, it would have no preference about whether it received \$5000 now or was paid by any of the 4 repayment plans.
- The 4 repayment plans are equivalent to one another and to \$5000 now at 8% interest

Economic equivalence allows us to compare alternatives on a common basis.

- Each alternative can be reduced to an *equivalent basis* dependent on
 - interest rate,
 - amount of money involved, and
 - timing of monetary receipts or expenses.
- Using these elements we can “move” cash flows so that we can compare them at particular points in time.