Question 1

(a) $|4+3x| \geq 10$.

soln.

$\Leftrightarrow 4+3x \leq -10$ or $4+3x \geq 10$.

for $4+3x \leq -10$,

$\frac{3x}{3} \leq \frac{-14}{3}$

$x \leq -\frac{14}{3}$.

for $4+3x \geq 10$,

$\frac{3x}{3} \geq \frac{6}{3}$

$x \geq 2$.

Therefore, $x \leq -\frac{14}{3}$ or $x \geq 2$

Answer: $(-\infty, -\frac{14}{3}] \cup [2, \infty)$

b) $|7-4x| > 3$

soln.

$\Leftrightarrow 7-4x < -3$ or $7-4x > 3$.

for $7-4x < -3$

$\frac{-4x}{-4} < \frac{-10}{-4} = 5/2$

$\therefore$ ~~$x < 1$~~ $x > 5/2$.

for $7-4x > 3$

$\frac{-4x}{-4} < \frac{-4}{-4}$ $-4x < -4$

$\therefore$ ~~$x > \frac{5}{4}$~~ $x < 1$

$\therefore x < 1$ or $x > 5/2$.

Answer: $(-\infty, 1) \cup (5/2, \infty)$

Question 2

$\frac{2x-7}{x-5} \leq 4$

Soln.

horizontal asymptote:

~~$2x-7=0$~~ $y=2$

~~H~~vertical asymptote: $x=5$.

Intercept: $\frac{2x-7}{x-5}=0$.

$x=3.5$ $(3.5, 0)$

y-intercept $y = -7/-5 = 7/5$

The region where the fxn ≤ 4 is:

$(-\infty, 5) \cup [13/2, \infty)$

Algebraically

Re-write the fxn in standard form:

$-\frac{2x+13}{x-5} \leq 0$.

$\therefore x-5 < 0$

$x < 5$

or

$-2x + 13 \leq 0$.

$\frac{2x}{2} = \frac{13}{2}$

$x \leq 13/2$.

$\therefore$ The interval is: $(-\infty, 5) \cup [13/2, \infty)$

Question 3.

(a) $F \circ G(x)$

soln.

$= f(2x-8)$

$= 2(2x-8)^2 - 3(2x-8) - 5.$

$= 2(2x^2 - 32x + 64) - (6x - 24) - 5.$

$= 8x^2 - 64x + 128 - 6x + 24 - 5$

$= 8x^2 - 64x - 6x + 128 + 24 - 5.$

$= 8x^2 - 70x + 147$

b) $G \circ F(x)$

$= G(2x^2 - 3x - 5)$

$= 2(2x^2 - 3x - 5) - 8.$

$= 4x^2 - 6x - 10 - 8$

$= 4x^2 - 6x - 18.$

$F(x) = x^3 + x^2 - 5x - 2.$

Soln.

$x^3 - x^2 - 5x - 2 = 0$

Factoring the equation;

Using the rational root theorem,

$$(x-2)\ \frac{x^3 - x^2 - 5x - 2}{x-2}$$

$$x-2 \;\overline{)\; x^3 - x^2 - 5x - 2}$$

$x^2 + 3x + 1$

$x^3 - 2x^2$

$-3x^2 - 5x - 2$

$3x^2 - 6x$

$x - 2$

$x - 2$

$\therefore$

$\therefore (x-2)(x^2 + 3x + 1) = 0.$

$\Rightarrow x - 2 = 0$ and $x = 2$

$x^2 + 3x + 1 = 0.$

$x = \frac{-3 \pm \sqrt{9-4}}{2}.$

$\therefore x = \frac{-3 \pm \sqrt{5}}{2}.$

Ans:

$x = 2,$

$x = \frac{-3 + \sqrt{5}}{2}$

$x = \frac{-3 - \sqrt{5}}{2}$

$\Rightarrow \left(2,\ \frac{-3+\sqrt{5}}{2},\ \frac{-3-\sqrt{5}}{2}\right)$

Question 5

$Q(x) = 5x - x^2$

(i) $Q(5+h)$

soln.

$= 5(5+h) - (5+h)^2$.

$= 25 + 5h - (25 + 10h + h^2)$

$= 25 + 5h - 25 - 10h - h^2$.

$= -5h - h^2$.

(ii) $Q(5)$

soln.

$= 5(5) - (5)^2$

$= 25 - 25 = 0$.

iii) $Q(5+h) - Q(5)$

$= (-5h - h^2) - 0$

$= -5h - h^2$.

iv) $\dfrac{Q(5+h) - Q(5)}{h}$, $h \neq 0$.

soln.

$\dfrac{-5h - h^2}{h} = \dfrac{h(-5-h)}{h}$.

$= -5 - h$

Question 6

a) $x^2 - 16 \geq 0$

soln.

Add 16 on both sides

$x^2 - 16 + 16 \geq 0 + 16$

$x^2 \geq 16.$

∴ square root both sides:

$x^2 \geq 16 \iff x \leq -\sqrt{16}$ or $x \geq \sqrt{16}.$

$\therefore x \leq -4$ or $x \geq 4$

$\therefore (-\infty, -4] \cup [4, \infty)$

b) $2x^2 + 11x < -14$

soln.

Add 14 on both sides

$2x^2 + 11x + 14 < 0.$

Factor.

$$\frac{-11 \pm \sqrt{121 - 112}}{4}$$

$$= \frac{-11 \pm 3}{4} \iff -2, -7/2.$$

$\therefore (x+2)(x+7/2) < 0$

$\iff -7/2 < x < -2$

Ans: $(-7/2, -2)$

Question 7 (A·15)

(a) $g(x) = \frac{4x+1}{3x-5}$.

Soln.

$y = \frac{4x+1}{3x-5}$.

$x = \frac{4y+1}{3y-5}$. $\Rightarrow (3y-5)x = 4y+1$

$= 3xy - 5x = 4y + 1$

$3xy - 4y = 5x + 1$

$y(3x-4) = 5x+1$

$\therefore y = \frac{5x+1}{3x-4}$. and $g^{-1}(x) = \frac{5x+1}{3x-4}$

b) $f(x) = \frac{4}{5x+2}$

Soln.

$y = \frac{4}{5x+2}$

$\therefore x = \frac{4}{5y+2}$. $\Rightarrow (5y+2)x = 4$

$5xy + 2x = 4$

$\frac{5xy}{5x} = \frac{4-2x}{5x}$.

$\therefore y = \frac{4-2x}{5x}$ and $f^{-1}(x) = \frac{4-2x}{5x}$.

$$\frac{3x^2+x-10}{8x^2+5x-14} \geq 0.$$

Soln.

factor the numerator:

$3x^2+x-10$

$$\frac{-1 \pm \sqrt{1+120}}{6} = \frac{-1 \pm 11}{6} = \frac{10}{6}, \frac{-12}{6}.$$

$\therefore \left(x-\frac{5}{3}\right)(x+2) = 0$

$\therefore x = 5/3$ and $x = -2.$

factor denominator.

$8x^2+5x-14$

$$\frac{-5 \pm \sqrt{25+56}}{2} = \frac{-5 \pm 9}{2} = \frac{4}{2}=2, \frac{-14}{2}=-7.$$

$\therefore 8(x-2)(x+7)$

Identify Intervals.

$x > 2$, or $x < -7$ or $-2 \leq x \leq 5/3.$

$\therefore$

Ans: $(-\infty, -7) \cup \left[-2, \frac{5}{3}\right] \cup (2, \infty)$

Question 9

$f(x) = -9x^2 + 12x$. ⇒ $\frac{f(x+h) - f(x)}{h}$.

$f(x+h) = -9(x+h)^2 + 12(x+h)$

$= -9(x^2 + 2xh + h^2) + 12x + 12h$.

$= -9x^2 - 18xh - 9h^2 + 12x + 12h$.

$= -9x^2 - 18xh + 12x + 12h - 9h^2$

$f(x+h) - f(x) = -9h^2 - 9x^2 - 18xh + 12x + 12h - (-9x^2 + 12x)$

$= -9x^2 + 18xh + 12x + 12h + 9x^2 - 12x - 9h^2$.

$= -18xh + 12h - 9h^2$.

$= h(-18x + 12 - 9h)$

$\therefore$

$\frac{f(x+h) - f(x)}{h} = \frac{h(-18x + 12 - 9h)}{h}$

$= -18x + 12 - 9h$.

Question 10

(a) If $f(x) = x^4 - 2x^2$.

soln.

A fxn is even when: $f(x) = f(-x)$

$f(-x) = (-x)^4 - 2(-x)^2$

$= x^4 - 2x^2$

$\therefore f(x) = f(-x)$

Hence the function is even.

b) $f(x) = x^2 + 8x$.

soln.

A fxn is even when: $f(x) = f(-x)$

$f(-x) = (-x)^2 + 8(-x)$

$= x^2 - 8x$

$\therefore f(x) \neq f(-x)$ and the fxn is not even.

A fxn is odd when:

$-f(x) = f(-x)$

$-f(x) = -(x^2 + 8x) = -x^2 - 8x$.

$f(-x) = x^2 - 8x$

$\therefore -f(x) \neq f(-x)$ and the fxn is not odd.

$\therefore$ The function is neither even nor odd.