Question 1

a) $x^2 - 3x \leq 4$

soln.

Subtract 4 from both sides.

$x^2 - 3x - 4 \leq 0$

factor the left side.

$$P = -4 \qquad N.o = \overset{+}{1}, \overset{+}{-4}$$
$$s = -3$$

$(x+1)(x-4) \leq 0$

$(x+1) \leq 0$ and $(x-4) \leq 0$

$\quad x \leq -1 \qquad$ and $x \leq 4.$ $(\Rightarrow)$ $-1 \leq x \leq 4$

Ans: $[-1, 4]$

b) $|2x - 7| \leq 15$.

soln.

$(\Rightarrow)$ $2x - 7 \leq 15$ and $2x - 7 \geq 15$.

solving $2x - 7 \leq 15$.

$\qquad 2x \leq 15 + 7$

$\qquad 2x \leq 22$

$\qquad x \leq 11$

solving $2x - 7 \geq -15$.

$\qquad \dfrac{2x}{2} \geq \dfrac{-8}{2}$

$2x \geq$ ⎯⎯⎯

$x \geq -4$

$\therefore -4 \leq x \leq 11$

Ans: $[-4, 11]$

a) $\dfrac{x+1}{x+2} > 5$

soln:

multiply both sides by $x+2$

$(x+1) > 5(x+2)$

$x+1 > 5x+10$ collect like terms together:

$x+1 = 5x+10$

$-4x = 9$

$x = -\dfrac{9}{4}$ Critical points: $x = -\dfrac{9}{4}, -2$.

Check intervals:

$x < -\dfrac{9}{4}$ doesn't work in original.

$x > -2$ doesnt work.

$\therefore$ Ans: $\left[-\dfrac{9}{4}, -2\right]$ $\left(-\dfrac{9}{4}, -2\right)$

b) $\dfrac{x+1}{x-2} \geq 3$

-soln:

multiply both sides by $(x-2)$

$x+1 = 3(x-2)$

$x+1 = 3x-6$

$\cancel{4} \; 2x = 7$

$x = \dfrac{7}{2}$.

$\therefore$ critical point:
$x = \dfrac{7}{2}, \quad x = 2$.

$\therefore \quad 2 < x \leq \dfrac{7}{2}$ since $\dfrac{7}{2}$ is included.

Ans: $\left(2, \dfrac{7}{2}\right]$

Question 3

(a) $f(x) = x^2 - 5x - 6$ and $g(x) = x - 5$.

a) $(f \circ g) x$.

$f(g(x)) = f(x^2 - 5)$

$\qquad = (x - 5)^2 - 5(x^2 - 5) - 6$

$\qquad = x^2 - 10x + 25 - 5x + 25 - 6$

$\qquad = x^2 - 15x + 44$

b) $(g \circ f)(x)$

$\qquad = g(x^2 - 5x - 6) = x^2 - 5x - 6 - 5$.

$\qquad = x^2 - 5x - 11$

Ans: $x^2 - 5x - 11$

Question 4

a) $(1-x)(x+4)(x-2) < 0$

soln.

$1-x < 0$

$1 < x \cdot \Rightarrow x > 1$

$x+4 < 0$

$x < -4. \quad \Rightarrow -4 > x.$

$(x-2) \leq 0 \quad \Rightarrow x = 2$

~~$x \leq 2$~~ $\Rightarrow x > 2$.

$\therefore \quad -4 < x < 1 \quad \text{or} \quad x > 2.$

Ans: $(-4, 1) \cup (2, \infty)$

b) $3x(x-3)(x+2) \geq 0$

sln

$3x(x-3)(x+2) = 0$

$3x = 0$

$x = 0$.

$x-3 = 0$.

$\Rightarrow x = 3$.

$x+2 = 0$.

$\Rightarrow x = -2$.

$\therefore \quad -2 \leq x \leq 0 \quad \text{or} \quad x \geq 3.$

Ans: $[-2, 0] \cup [3, \infty)$

Question 5

$$f(x) = \sqrt{7x + 4}$$

a) $f(0) = \sqrt{7(0) + 4}$ $= \sqrt{4}$ $= \pm 2$ Ans: ± 2.

b) $f(4) = \sqrt{(7(4) + 4)}$ $= \sqrt{32}$ $= \pm \sqrt{16 \times 2} = \pm 4\sqrt{2}$.

c) $f(3) = \sqrt{7(3) + 4}$ $= \sqrt{25}$ $= \pm 5$ Ans: ± 5

d) $f(2y) = \sqrt{7(2y) + 4}$ $= \sqrt{14y + 4}$

e) $f(t-1) = \sqrt{7(t-1) + 4}$ $= \sqrt{.7t - 7 + 4}$

    Ans $= \sqrt{7t - 3}$

$$g(x) = \frac{x-4}{x+3}$$

a) $g(5) = \dfrac{5-4}{5+3} = \dfrac{1}{8}$ Ans $= \dfrac{1}{8}$

b) $g(4) = \dfrac{4-4}{4+3} = \dfrac{0}{7} = 0$ Ans: 0

c) $g(-3) = \dfrac{-3-4}{-3+3} = \dfrac{-7}{0} =$ undefined.   Ans: undefined

d) $g(-16.25) = \dfrac{-16.25-4}{-16.25+3} = \dfrac{-20.25}{-13.25} = \dfrac{81}{53}$

Ans: $\dfrac{81}{53}$ OR 1.6283

e) $g(x+h) = \dfrac{x+h-4}{x+h+3}$

Ans: $\dfrac{x+h-4}{x+h-3}$.

Question 7

$$|3x-4| \leq 2x+1$$

   soln:

$\Rightarrow$ $3x-4 \leq 2x+1$ or $3x-4 \geq -2x-1$.

solving $3x-4 \leq 2x+1$

$$x \leq 5.$$

solving $3x-4 \geq -2x-1$

$$3x+2x \geq -1+4$$

$$5x \geq 3$$

$$x \geq \tfrac{3}{5}.$$

$(\Rightarrow)$ $\tfrac{3}{5} \leq x \leq 5$.

Ans: $\left[\tfrac{3}{5}, 5\right]$

Question 8

a) $|6x - 3| = 15$.

   soln·

$6x - 3 = 15$ or $6x - 3 = -15$.

solving $6x - 3 = 15$·

$\frac{6x}{6} = \frac{18}{6}$

$x = 3$·

solving $6x - 3 = -15$·   we get $\frac{6x}{6} = \frac{-12}{6}$·

$x = -2$.

∴ $x = -2$ or 3·

b) $(x - 4)(x + 3) > (x - 1)^2$

   soln·

$x(x + 3) - 4(x + 3) \geq x^2 - 2x + 1$

$x^2 + 3x - 4x + 12 \geq x^2 - 2x + 1$

$x^2 - x + 12 \geq x^2 - 2x + 1$

⟹ $x - 13 > 0$·

⟹ $x > 13$..

Ans: $(13, \infty)$

Question 9

$$\frac{x^2 - x - 6}{x^2 + 4x - 5} \leq 0.$$

soln:

factor: $x^2 - x - 6$ P= -6, S = -1   N.o: 2, -3.

$x^2 + 4x - 5$ P = -5, S = 4, N.o = -1, 5.

$\therefore \frac{(x+2)(x-3)}{(x-1)(x+5)} \leq 0.$

critical points: $x = 1$, $x = -5$.

$(x+2) = 0$ $\Rightarrow x = -2.$

$(x-3) = 0$ $\Rightarrow x = 3.$

$\therefore -5 \leq x \leq -2$ or $1 < x \leq 3$

Ans: $(-5, -2] \cup (1, 3]$

Question 10.

$$|x^2 + 3x - 1| < 3$$

soln.

(⇒) $-3 < x^2 + 3x - 1 < 3$.

(⇒) $x^2 + 3x - 1 > -3$ and $x^2 + 3x - 1 < 3$.

solving $x^2 + 3x - 1 > -3$.

$x^2 + 3x + 2 \geq 0$ P= 2 S=3   N.O= 2, 1

$(x + 2)(x + 1) \geq 0$.

$x + 2 = 0$ (⇒) $x < -2$.   (⇒) $x < -2$ or $x < -1$

$x + 1 = 0$ (⇒) $x < -1$

solving $x^2 + 3x - 1 < 3$.

$x^2 + 3x - 4 \geq 0$ P= -4, S = 3 N.O= -1, 4

$(x - 1)(x + 4) \geq 0$.

$x - 1 = 0$ (⇒) $x < 1$ (⇒) $-4 < x < 1$

$x + 4 = 0$ (⇒) $x > 4$

Merging overlapping Intervals:

$-4 < x < -2$ or $-1 < x < 1$

Ans: $(-4, -2) \cup (-1, 1)$