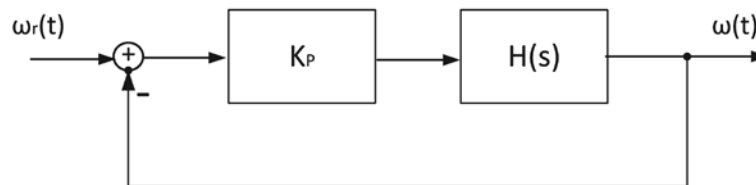


Exercise Package 4:

Laplace transform:

- 1) Given the unit impulse response of a LTI system, find its transfer function $H(s)=B(s)/A(s)$ in canonical form and ROC using the definition of Laplace transform and state the stability and causality with a specific reason:
 - a. $h_a(t)=600e^{-30t}u(t)$
 - b. $h_b(t)=600(e^{-30t}+e^{-36000t})u(t)$
 - c. $h_c(t)=600e^{-30t}u(t-0.4)$
 - d. $h_d(t)=600e^{30t}u(-t)$
 - e. $h_e(t)=600e^{-30t}[u(t)-u(t-2)]$
 - f. $h_f(t)=h_a(0.2t)$ and show that $H_f(s)=(1/0.2)H_a(s/0.2)$
 - g. A practical Butterworth filter, $h_g(t)=10198e^{-3214t}\sin(3214t)u(t)$ (Tip: $\sin(\omega)=(e^{j\omega}-e^{-j\omega})/(2j)$)
 - h. $h_h(t)=600te^{-30t}u(t)$ (Tip: integral by parts $\int u dv = uv - \int v du$)
- 2) Given the unit impulse response of a LTI system, find the bound of its absolute integration, if it is bounded, $\int_0^\infty |h(t)|dt \leq M < \infty$.
 - a. A practical integrator, $h_a(t)=-1000e^{-100t}u(t)$
 - b. A field controlled DC motor, $h_b(t)=1100(e^{-2t}+e^{-40000t})u(t)$
 - c. A second order filter, $h_c(t)=3200e^{-16t}\cos(40t)u(t)$
- 3) Find the unit impulse response of the following transfer functions. All the systems are causal, that is, $h(t)=0$, for $t<0$ or equivalently, R.O.C. $\sigma>\alpha$ (the largest real part of the poles, $A(s)=0$). Point out the stability of each system from the location of their poles and ROC.
 - a. $H_a(s) = \frac{600}{s+30}$
 - b. $H_b(s) = \frac{32768530}{s^2+6428s+20661116}$
 - c. $H_c(s) = \frac{3s+2}{s^2+6s+8}$
 - d. $H_d(s) = \frac{s^2+6s+1}{s^2+s+16}$
 - e. $H_e(s) = \frac{s^2+6s+1}{s^3+s^2+16s}$
- 4) Practices of using Laplace transform table and Laplace transform properties to solve the following problems:
 - a. $h_a(t)=2e^{-3(t-2)}u(t)$ find Laplace transform $H_a(s)$.
 - b. $h_b(t)=2e^{-3t}u(t-2)$ find Laplace transform $H_b(s)$.
 - c. $h_c(t)=2e^{3t}u(-t-2)$ find Laplace transform $H_c(s)$.
 - d. $h_d(t)=te^{-3t}u(t-2)$ find Laplace transform $H_d(s)$.
 - e. $H_e(s) = \frac{6s+1}{e^{2s+0.5}(s^2+5s+6)}$ find unit impulse response $h_e(t)$.
- 5) For a system with transfer function $H(s) = \frac{2s+5}{s^2+9s+20}$, find the zeros state response of the following input signals using Laplace transform:
 - a. $2e^{-3t}u(t)$
 - b. $2e^{-3t}u(t-2)$ (Tip: Apply the properties of Laplace transform and Use the answer in 5a))
 - c. $2e^{-3(t-2)}u(t)$
 - d. $2e^{-4t}u(t)$
- 6) The transfer function of a DC motor is $H(s) = \frac{2000}{s+25}$ (radian/(Volt•second)).

- a. Find the Laplace transform of zero state response, $\Omega_{zs}(s)$, to the input signal $4u(t)$ (V).
 - b. Using final value theorem to find its steady state speed, $\omega_{ss}(t)$, that is, $\lim_{t \rightarrow \infty} \omega(t)$.
 - c. Find its time domain expression of the zero state response, $\omega_{zs}(t)$, using inverse Laplace transform.
 - d. Find its time domain expression of the zero state response, $\omega_{zs}(t)$, to the input signal $4[u(t)-u(t-50\text{mS})]$ using inverse Laplace transform.
 - e. Find its time domain expression of the zero state response, $\omega_{zs}(t)$, to the input signal $4[u(t)-u(t-50\text{mS})]+4u(t-100\text{mS})$ using inverse Laplace transform. (Tip: The answers of this problem should match the answers from exercise package 3 problem 2).)
- 7) The transfer function of a coupling network is $H(s) = \frac{0.916s}{s+7331}$. Use Laplace transform to find its zero state response to the input signal $v_{in}(t)=0.2[u(t)-u(t-0.0005)]$ (1KHz 50% duty-cycle square wave).
- 8) The system diagram of a proportional feedback control of motor speed is shown below:



The transfer function for the DC motor with load is $H(s) = \frac{1000}{s+1}$ which has time constant 1 second and rise time (90%) 2.3 second. $\omega_r(t)$ is required speed (controlled speed).

- a. Find the closed loop transfer function $H_c(s)$, ($\triangleq \frac{\omega(t)}{\omega_r(t)}$).
 - b. Use Laplace transform to solve the zero state response to the required speed $\omega_r(t)=100u(t)$ (radian/second) with proportional control coefficient (gain) $K_p=0.01$. Answer the following questions:
 - i. What is the steady state speed of the DC motor and the error from the required speed?
 - ii. What is the rise time (90%)? Compare with the DC motor without a feedback controller.
 - c. Use Laplace transform to solve the zero state response to the required speed $\omega_r(t)=100u(t)$ (radian/second) with proportional control coefficient (gain) $K_p=0.1$ and 1 respectively. Answer the following questions:
 - i. Explain the steady state speed error with respect to the proportional control coefficient K_p .
 - ii. Explain the change of rise time with respect to the proportional control coefficient K_p .
 - d. Use Laplace transform to solve the zero state response to the required speed $\omega_r(t)=100u(t)-50u(t-0.1)$ (radian/second) with proportional control coefficient (gain) $K_p=0.01$ and 1 respectively. Plot the responses with the required speed (input) and comment on your result.
- 9) The transfer function of a DC motor is $H(s) = \frac{20000000}{(s+25)(s+10000)}$ (radian/(Volt•second)).
- a. Find the Laplace transform of zero state response, $\Omega_{zs}(s)$, to the input signal $4u(t)$ (V).
 - b. Using final value theorem to find its steady state speed, $\omega_{ss}(t)$, that is, $\lim_{t \rightarrow \infty} \omega(t)$.

- c. Find its time domain expression of the zero state response, $\omega_{zs}(t)$, using inverse Laplace transform.
- d. Find its time domain expression of the zero state response, $\omega_{zs}(t)$, to the input signal $4[u(t)-u(t-50\text{mS})]+4u(t-100\text{mS})$ using inverse Laplace transform. Use matlab to plot the output motor speed $\omega_{zs}(t)$ for $0 \leq t \leq 200 \times 10^{-3}$ second. Compare the result with respect the answer from problem 6. (Tip: Add the speed curve to the figure plotted in problem 6. You will see they are very close to each other. This is the reason that we can do model order reduction by removing the poles at a very high value (not dominant poles).)